

High School Math Summer Packet 2026

This packet provides practice on prerequisite skills needed for the Enhanced 8th/Algebra 1 and Advanced Algebra 1 courses.

Multistep Equations Notes	p. 1-3
Multistep Equations Practice and answer key	p. 4-5
Literal Equations Notes	p. 6
Literal Equations Practice and answer key	p. 7-8
Inequalities Notes	p. 9
Inequalities Practice and answer key	p. 10-11
Functions Notes	p. 12-15
Functions Practice and answer key	p. 16-17
Linear Functions Notes	p. 18-22
Linear Functions Practice and answer key	p. 23-26
Systems of Linear Equations Notes	p. 27
Systems of Linear Equations Practice and answer key	p. 28-29
Laws of Exponents Notes	p. 30
Laws of Exponents Practice and answer key	p. 31-35

Name: _____

SOLVING EQUATIONS WITH VARIABLES ON BOTH SIDES!

- Step 1: Distribute . (on each side of = sign)
- Step 2: Combine like terms. (on each side of = sign)
- Step 3: Move all variable terms to one side. Using inverse operations
- Step 4: Solve the remaining two-step equation.

EXAMPLES:

$$\begin{array}{r} 5y - 8 = 3y + 12 \\ -3y \quad -3y \\ \hline 2y - 8 = 12 \\ +8 \quad +8 \\ \hline 2y = 20 \\ \frac{2y}{2} = \frac{20}{2} \\ y = 10 \end{array}$$

$$\begin{array}{r} 6x - 10 = \frac{1}{2}(7x - 6) \\ \frac{6}{1} = \frac{12}{2} \\ \frac{-7}{2} = \frac{-7}{2} \\ \hline \frac{5}{2} \\ \hline 6x - 10 = \frac{7}{2}x - 3 \\ -\frac{7}{2}x \quad -\frac{7}{2}x \\ \hline \frac{5}{2}x - 10 = -3 \\ +10 \quad +10 \\ \hline \frac{2}{5} \cdot \frac{5}{2}x = 7 \cdot \frac{2}{5} \\ x = \frac{14}{5} \text{ or } 2.8 \end{array}$$

Solving Multi-Step Equations

Dealing with Fractions

There are two different methods for simplifying an equality with fractions.

Method 1	Method 2
Finding a Common Denominator	Multiplying out the fractions

How are methods 1 and 2 alike?

They both help you eliminate the fractions so you can solve for X.

Which method do you prefer?

Personally, I prefer method 2 because there are no fractions to subtract or add.

Method 1) $X=24$

$$\frac{3x}{4} - \frac{x}{3} = 10$$

$$\frac{3x \cdot 3}{4 \cdot 3} - \frac{x \cdot 4}{3 \cdot 4} = 10$$

$$\frac{9x}{12} - \frac{4x}{12} = 10$$

$$12 \cdot \frac{5x}{12} = 10 \cdot 12$$

$$\frac{5x}{5} = \frac{120}{5}$$

$$x = 24$$

$\frac{3}{3}$ and $\frac{4}{4} = 1$,
so we don't have to multiply both sides of the equals sign.

check:

$$\frac{3(24)}{4} - \frac{24}{3} = 10$$

$$\frac{72}{4} - 8 = 10$$

$$18 - 8 = 10$$

$$10 = 10 \checkmark$$

Method 2) $X=24$

$$\frac{3x}{4} - \frac{x}{3} = 10$$

$$4 \cdot 3 \cdot \left(\frac{3x}{4} - \frac{x}{3} = 10 \right)$$

$$12 \cdot \left(\frac{3x}{4} - \frac{x}{3} \right) = 12 \cdot 10$$

$$\frac{36x}{4} - \frac{12x}{3} = 120$$

$$9x - 4x = 120$$

$$\frac{5x}{5} = \frac{120}{5}$$

$$x = 24$$

Got it) How can you find the least common multiple of 5 and 4? How does finding the least common multiple (LCM) help you solve the equation?

a) You could multiply $5 \cdot 4$.

b) You could multiply both sides of the equation by the LCM in Method 2 to eliminate the fractions in the problem.

1-4 Solving Multi-Step Equations TEKS A.5(A)

Steps:

1. Distribute. (remember to be careful of negative signs)
2. Combine Like Terms
3. + or - the smaller variable term over
4. + or - the constant on both sides.
5. x or ÷ the coefficient on both sides.
6. Check your answer

Examples

$$\begin{aligned}
 2(x-8) + 7 &= 5(x+2) - 3x - 19 \\
 2x - 16 + 7 &= 5x + 10 - 3x - 19 \\
 2x - 9 &= 2x - 9 \\
 \cancel{-2x} \quad \quad \quad \cancel{-2x} & \\
 -9 &= -9
 \end{aligned}$$

All real numbers
or
Infinite Solutions

← Special cases →

$$\begin{aligned}
 3(-2 - 3x) &= -9x - 4 \\
 -6 - 9x &= -9x - 4 \\
 +9x \quad +9x & \\
 -6 &\neq -4
 \end{aligned}$$

No Solution

$$\frac{x-8}{12} \quad \cancel{\neq} \quad \frac{2x+5}{3}$$

$$\begin{aligned}
 3(x-8) &= 12(2x+5) \\
 3x - 24 &= 24x + 60 \\
 \cancel{-3x} \quad \quad \quad \cancel{-3x} & \\
 -24 &= 21x + 60 \\
 \cancel{-60} \quad \quad \quad \cancel{-60} & \\
 -84 &= 21x \\
 \frac{-84}{21} &= \frac{21x}{21} \\
 -4 &= x
 \end{aligned}$$

$-4 = x$

Five times the sum of (a number and three) is the same as three multiplied by one less than twice the number. What is the number?

$$\begin{aligned}
 5(x+3) &= 3(2x-1) \\
 5x + 15 &= 6x - 3 \\
 \cancel{-5x} \quad \quad \quad \cancel{-5x} & \\
 15 &= x - 3 \\
 +3 \quad \quad \quad +3 & \\
 18 &= x
 \end{aligned}$$

$18 = x$

Summary:

Solving Multistep Equations

Name _____

Date _____ Period _____

Solve each equation.

1) $-2(5a + 7) = -94$

2) $-2x - 3(2 - 4x) = -86$

3) $-10 - 3n = 8(2 - 4n) + 3$

4) $-7(1 + 6m) = -7 + 7m$

5) $14 + 3p = 3(p + 5)$

6) $35 - 5r = -5(r - 7)$

7) $-\frac{8}{3}p - \frac{5}{3}\left(\frac{5}{3}p + 1\right) = -\frac{145}{12} + \frac{3}{2}p$

8) $-\frac{71}{12} + \frac{1}{2}p = 2\left(-\frac{4}{3}p + 1\right)$

9) $2.2v - 5.74 = 2.5 - 2.5(v + 0.1)$

10) $10.2 + 0.8v = -1.7 + 2.7(v + 3)$

11) $0.5(0.5 + 0.6m) = -1.9(m + 2.3)$

12) $-3(1 - 2.2m) = 3(2.4m - 0.9)$

Answers to Solving Multistep Equations

1) $\{8\}$

5) No solution.

8) $\left\{\frac{5}{2}\right\}$

12) $\{-0.5\}$

2) $\{-8\}$

6) $\{\text{All real numbers.}\}$

9) $\{1.7\}$

3) $\{1\}$

10) $\{2\}$

7) $\left\{\frac{3}{2}\right\}$

4) $\{0\}$

11) $\{-2.1\}$

1-5 Solving Literal Equations TEKS A.12(E)

Steps:

Same steps as multi-step equations. Except that most of the time you do not want to distribute. That will only make the problem harder.

Examples

$$2 \cdot A = \frac{h(a+b)}{2} \text{ Solve for h.}$$

$$\frac{2A}{(a+b)} = \frac{h(a+b)}{(a+b)}$$

$$H = \frac{2A}{a+b}$$

$$P = 2L + 2W \text{ Solve for W.}$$

$$-2L -2L$$

$$\frac{P-2L}{2} = \frac{2W}{2}$$

$$\frac{P}{2} - L = W$$

$$\frac{x(4-k)}{x} = \frac{P}{x} \text{ Solve for k.}$$

$$\frac{4-k}{-4} = \frac{P}{x} - 4$$

$$\frac{-k}{-1} = \frac{P}{x} - 4$$

$$k = -\frac{P}{x} + 4$$

$$SA = B + \frac{1}{2} PL \text{ Solve for P.}$$

$$-B -B$$

$$2(SA-B) = \frac{1}{2} PL \cdot \frac{2}{1}$$

$$\frac{2(SA-B)}{L} = \frac{PL}{L}$$

$$\frac{2(SA-B)}{L} = P$$

or

$$\frac{2SA - 2B}{L} = P$$

Summary:

LESSON
2-3

Solving for a Variable (Literal Equations)

Reteach

Solving for a variable in a formula can make the formula easier to use.

You can solve a formula, or literal equation, for any one of the variables.

To solve a literal equation or formula, underline the variable you are solving for, and then **undo** what has been done to that variable. Use inverse operations in the same way you do when solving an equation or inequality.

The formula for finding the circumference of a circle when you know diameter is $C = \pi d$. If you know the circumference, you could find the diameter by using a formula for d .

Examples

Solve $C = \pi d$ for d .

$$\frac{C}{\pi} = \frac{\pi d}{\pi}$$

$$\frac{C}{\pi} = d \text{ or } d = \frac{C}{\pi}$$

Since d is multiplied by π , use division to **undo** this.

Divide both sides by π .

Solve $F = \left(\frac{9}{5}\right)C + 32$ for C .

$$F - 32 = \left(\frac{9}{5}\right)C + 32 - 32$$

$$\left(\frac{5}{9}\right)(F - 32) = \left(\frac{5}{9}\right)\left(\frac{9}{5}\right)C$$

$$\left(\frac{5}{9}\right)(F - 32) = C$$

What has been done to C ? First **undo** adding 32.

Subtract 32 from both sides.

Multiply both sides by $\left(\frac{5}{9}\right)$, the reciprocal of $\left(\frac{9}{5}\right)$.

Simplify.

Solve each formula for the indicated variable.

1. $A = \frac{1}{2}bh$, for b

2. $A = lw$, for l

3. $R = \frac{2s - 6t + 5}{2}$, for s

4. $P = a + b + c$, for c

5. $l = \frac{1}{2}prt$, for t

6. $G = \frac{HJ}{K}$, for H

Solve each equation for the indicated variable.

7. $m + n + p + q = 360$, for n

8. $t = rs + s$, for s

9. $\frac{a}{b} = \frac{c}{d}$, for a

Solving for a Variable Answer Key

$$\textcircled{1} \cdot A = \frac{1}{2}bh \text{ for } b$$

$$\frac{2A}{h} = \frac{bh}{h}$$

$$\frac{2A}{h} = b$$

$$\textcircled{2} \cdot A = \frac{lw}{w} \text{ for } l$$

$$\frac{A}{w} = l$$

$$\textcircled{3} \cdot R = \frac{2s - 6t + 5}{2} \text{ for } s$$

$$2R = 2s - 6t + 5$$

$$2R \stackrel{-5}{=} 2s - 6t$$

$$\frac{2R - 5 + 6t}{2} = \frac{2s}{2}$$

$$\frac{2R - 5 + 6t}{2} = s$$

$$\textcircled{4} \cdot P = a + b + c \text{ for } c$$

$$P - a = b + c$$

$$P - a - b = c$$

$$\textcircled{5} \cdot I = \frac{1}{2}prt \text{ for } t$$

$$\frac{2I}{pr} = \frac{prt}{pr}$$

$$\frac{2I}{pr} = t$$

$$\textcircled{6} \cdot G = \frac{HJ}{K} \text{ for } H$$

$$\frac{KG}{J} = \frac{HJ}{J}$$

$$\frac{KG}{J} = K$$

$$\textcircled{7} \cdot m + n + p + q = 360$$

$$n = 360 - m - p - q$$

$$\textcircled{8} \cdot t = rS + S$$

$$\frac{t}{r+1} = \frac{S(r+1)}{r+1}$$

$$\frac{t}{r+1} = S$$

$$\textcircled{9} \cdot \frac{a}{b} = \frac{c}{d} \text{ for } a$$

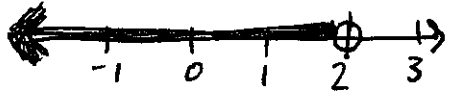
$$a = \frac{cb}{d}$$

1-6 Solving Inequalities TEKS A.5(B)


Rules:

Flip the inequality sign if you multiply or divide by a negative coefficient.
 Open dot for $<$ or $>$ Closed dot for \leq or \geq
 Color to the left for $<$ or \leq and color to the right for $>$ or \geq

Examples

$$\begin{aligned}
 3d - 2(8d - 9) &> -2d - 4 \\
 3d - 16d + 18 &> -2d - 4 \\
 -13d + 18 &> -2d - 4 \\
 +2d \quad \quad +2d & \\
 -11d + 18 &> -4 \\
 -18 \quad -18 & \\
 \frac{-11d}{-11} > \frac{-22}{-11} & \quad * \text{ flip sign } * \\
 d < 2
 \end{aligned}$$


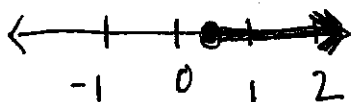
A number line with arrows at both ends, labeled from -1 to 3. An open circle is drawn at the number 2, and the line is shaded to the left of this circle.

$$\begin{aligned}
 -2x - 2(3x + 3) &\leq 66 \\
 -2x - 6x - 6 &\leq 66 \\
 -8x - 6 &\leq 66 \\
 +6 \quad +6 & \\
 -8x &\leq 72 \\
 \frac{-8x}{-8} &\frac{72}{-8} \\
 x &\geq -9
 \end{aligned}$$


A number line with arrows at both ends, labeled from -10 to -6. A closed circle is drawn at the number -9, and the line is shaded to the right of this circle.

Negative sixteen increased by $36x$ is at least $16x$ less than ten.

$$\begin{aligned}
 -16 + 36x &\geq 10 - 16x \\
 \quad +16x \quad &\quad +16x \\
 -16 + 52x &\geq 10 \\
 +16 \quad \quad +16 & \\
 52x &\geq 26 \\
 \frac{52x}{52} &\frac{26}{52} \\
 x &\geq \frac{1}{2}
 \end{aligned}$$



Symbol	Name	Words
$<$	Less Than	Is under, fewer, below
$>$	Greater Than	Is more than, exceeds, is over
\leq	Less than or equal to	At most, no more than, maximum.
\geq	Greater than or equal to	At least, not less than, is not under. minimum

Summary:

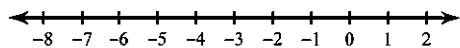
Inequalities

Name _____

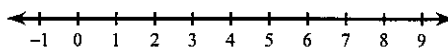
Date _____ Period _____

Solve each inequality and graph its solution.

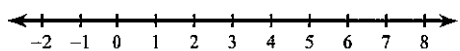
1) $9 < -7r - 2r$



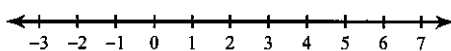
2) $7 > 2x - x$



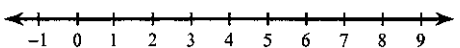
3) $-3r - 3 \geq -3 - 3r$



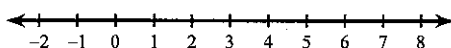
4) $4 - 7n \leq -4n - 8$



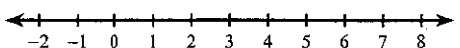
5) $-1 - 3r > -3r - 1$



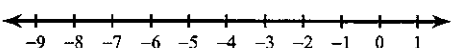
6) $8(6p - 7) \leq -15 + 7p$



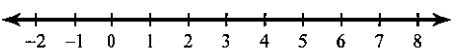
7) $3(2 + 7m) < 6 + 4m$



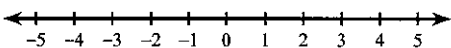
8) $-5n - 4(6n + 8) < 5n + 2$



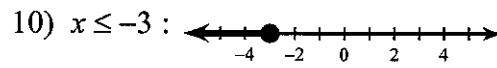
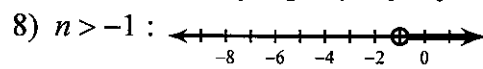
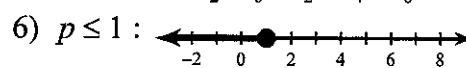
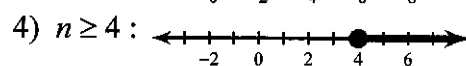
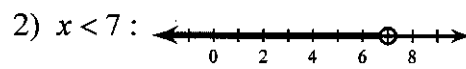
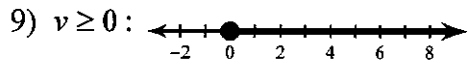
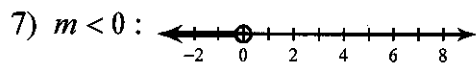
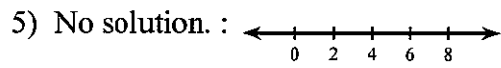
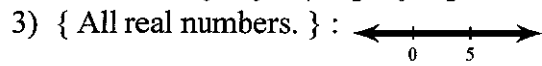
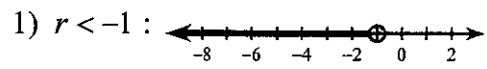
9) $-\frac{18}{35} + \frac{2}{3}v \leq v + \frac{9}{7}\left(\frac{3}{2}v - \frac{2}{5}\right)$



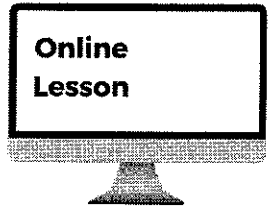
10) $4.2(1 + 0.8x) + 4.1 \leq 3.1x + 7.52$



Answers to Inequalities



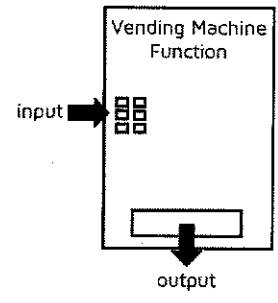
Intro to Functions



What is a Function?

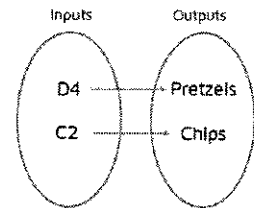
A function is a lot like a vending machine.
 The code you type in is the input and the food/drink you get out is the output.

Definition: A function is a relationship where each input has one output.

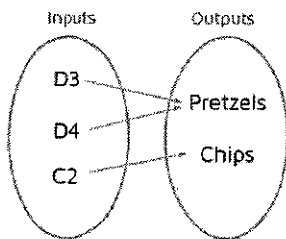


Mapping Diagrams

The inputs are written on the left side and the outputs are written on the right side. Arrows show which input and output values go together.

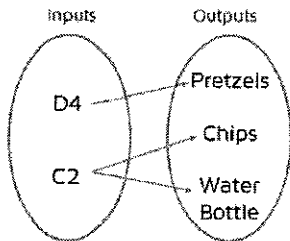


Does this mapping represent a function? Why or why not?



Yes. There can be more than one input that corresponds to the same output.

Does this mapping represent a function? Why or why not?



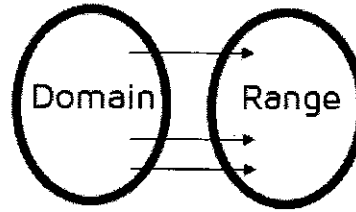
No. The input C2 has been assigned to two different output values.

A function can only assign an input value to one output value.

Domain and Range

Domain: set of all input values

Range: set of all output values



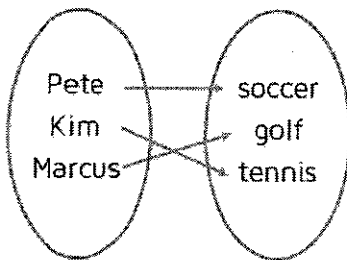
For the vending machine example,

Domain = { D3, D4, C2 }

Range = { pretzels, chips }

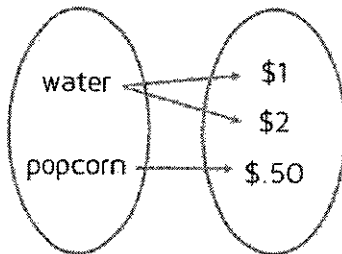
Practice

1. Is this a function? Why or why not?



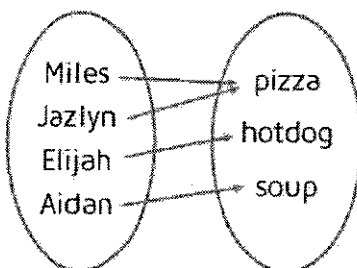
Yes. Each input has been assigned to only one output (only one sport was assigned to each student).

Is this a function? Why or why not?



No. The water has been assigned to two different outputs: \$1 and \$2. An input can only be assigned to one output.

Is this a function? Why or why not?



Yes. Each input has been assigned to only one output. It's ok if the same output is used more than once.

Function Notation

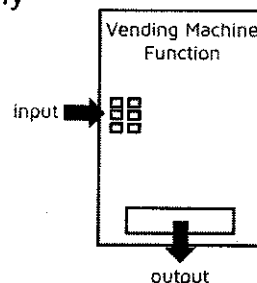


What are Functions?

A function takes an input value and assigns it exactly one output value.

List 4 ways to represent a function:

- 2) Mapping 2) Table
 4) Graph 4) Ordered Pairs



Functions as Equations

An equation tells you exactly what the function is going to do with the input value to get the output value.

The function $y = x - 1$ takes an input value and subtracts 1 to give the output values.

The function $y = x/2$ divides input values by 2 to give the output values.

Function	What does it do?	When $x = 9$, $y = ?$
$y = 4x$	Multiplies the input values by 4	$y = 4(9) = 36$
$y = x + 3$	Adds 3 to the input values	$y = 9 + 3 = 12$
$y = \sqrt{x}$	Takes the square root of the input values	$y = \sqrt{9} = 3$

Function Notation

The most popular letter used to name a function is the letter f, but about any letter can be used.

If $f(x) = 2x$, f is the name of the function, x is the input, and the $2x$ tells you what the function does (multiplies input values by 2).

$f(x) = 2x$ is the same thing as $y = 2x$.

Function notation allows us to give the function a name.

Written as an Equation	Written as a Function
$y = 5x$	$f(x) = 5x$
$y = x + 7$	$g(x) = x + 7$
$y = x^2$	$h(x) = x^2$
$y = 2x - 6$	Sample: $f(x) = 2x - 6$

The most common input variable is x, but other letters and symbols can be used.

True or False? $f(x) = 3x - 1$ is the same as $f(a) = 3a - 1$ True

True or False? $f(x)$ means f times x . False

$h(a)$ means h is the name of the function and a is the variable used for the input values.

Evaluating Functions

What does $f(8)$ mean? We need to plug in 8 for the input value.

If $f(x) = x - 3$, then $f(8) = \underline{8 - 3 = 5}$

Function	Evaluate $f(4)$
$f(x) = 5x$	$5(4) = 20$
$f(t) = t + 2$	$4 + 2 = 6$
$f(a) = a^2 + 3a + 1$	$4^2 + 3(4) + 1 = 29$
$f(x) = 4x - 3$	$4(4) - 3 = 13$

If the function has more than one term with a variable, make sure to plug your input value into each one.

Practice

- Does $f(x)$ mean f times x ? No
- $f(x) = 3x + 2$ is the same as $y = 3x + 2$ Yes
- If $f(x) = 4x + 8$, $f(5) = \underline{4(5) + 8 = 28}$
- If $g(t) = 2t - 6$, $g(10) = \underline{2(10) - 6 = 14}$

Relations and Functions Practice

1. Write the domain and range of each set of ordered pairs. Remember to write your answers in the {} symbols, order from least to greatest, and you do not have to write repeated numbers more than once.

a. $\{(3, -3), (6, -3), (9, -3), (12, -3)\}$

b. $\{(7, 4), (7, 5), (-3, 4), (-1, 5)\}$

c. $\{(3, 0), (7, 1), (4, 6), (3, 8)\}$

Domain _____

Domain _____

Domain _____

Range _____

Range _____

Range _____

Function: yes or no

Function: yes or no

Function: yes or no

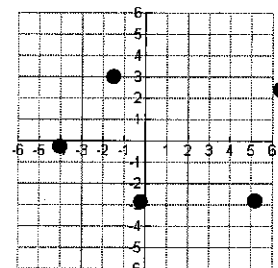
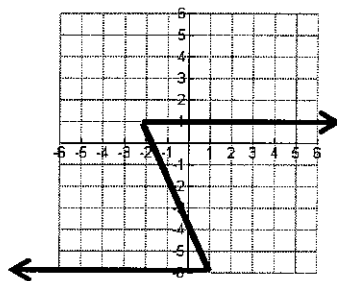
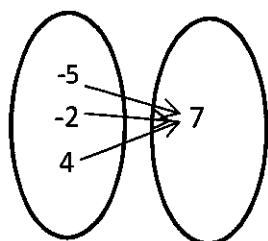
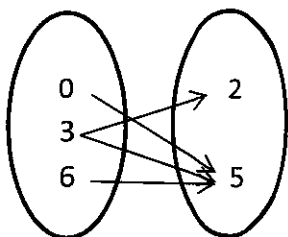
2. Is the relation a function? Circle yes or no.

a. yes or no

b. yes or no

c. yes or no

d. yes or no



e. Is the table a function?

yes or no

x	y
7	-8
6	-9
5	-10
6	-11
5	-12

For # 6– 8 Evaluate each function for the given INPUT values.

3. $f(x) = 7x - 3$; Find $f(8)$

4. $h(x) = \frac{1}{4}x - 6$; Find $h(12)$

5. $f(x) = x^2 + 1$; Find $f(-2)$

For # 9-11 Evaluate each function for the given OUTPUT values.

6. $S(t) = 9t - 4$

Find t if $S(t) = 32$

7. $j(x) = 2x + 9$

Find x if $j(x) = 23$

8. $f(x) = \frac{1}{2}x + 3$

Find x if $f(x) = 11$

Relations and Functions Answer Key

1. Write the domain and range of each set of ordered pairs. Remember to write your answers in the {} symbols, order from least to greatest, and you do not have to write repeated numbers more than once.

a. $\{(3, -3), (6, -3), (9, -3), (12, -3)\}$

Domain $\{3, 6, 9, 12\}$

Range $\{-3\}$

Function: yes or no

b. $\{(7, 4), (7, 5), (-3, 4), (-1, 5)\}$

Domain $\{-3, -1, 7\}$

Range $\{4, 5\}$

Function: yes or no

c. $\{(3, 0), (7, 1), (4, 6), (3, 8)\}$

Domain $\{3, 4, 7\}$

Range $\{0, 1, 6, 8\}$

Function: yes or no

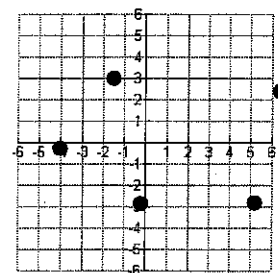
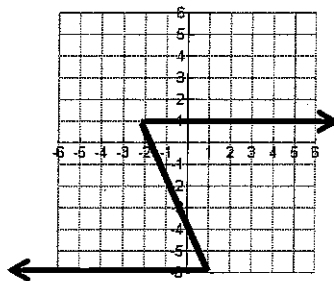
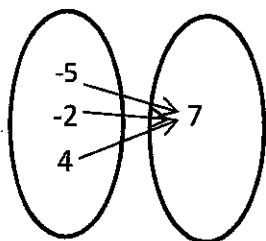
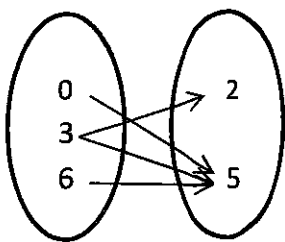
2. Is the relation a function? Circle yes or no.

a. yes or no

b. yes or no

c. yes or no

d. yes or no



e. Is the table a function?

yes or no

x	y
7	-8
6	-9
* 5	-10
6	-11
* 5	-12

For # 6-8 Evaluate each function for the given INPUT values.

3. $f(x) = 7x - 3$; Find $f(8)$

$$\begin{aligned} &= 7(8) - 3 \\ &= 56 - 3 \\ &= 53 \end{aligned}$$

4. $h(x) = \frac{1}{4}x - 6$; Find $h(12)$

$$\begin{aligned} &= \frac{1}{4}(12) - 6 \\ &= 3 - 6 \\ &= -3 \end{aligned}$$

5. $f(x) = x^2 + 1$; Find $f(-2)$

$$\begin{aligned} &= (-2)^2 + 1 \\ &= 4 + 1 \\ &= 5 \end{aligned}$$

For # 9-11 Evaluate each function for the given OUTPUT values.

6. $S(t) = 9t - 4$

Find t if $S(t) = 32$

$$\begin{aligned} 32 &= 9t - 4 \\ +4 & \quad +4 \\ 36 &= 9t \\ \frac{36}{9} &= \frac{9t}{9} \\ 4 &= t \end{aligned}$$

7. $j(x) = 2x + 9$

Find x if $j(x) = 23$

$$\begin{aligned} 23 &= 2x + 9 \\ -9 & \quad -9 \\ 14 &= 2x \\ \frac{14}{2} &= \frac{2x}{2} \\ 7 &= x \end{aligned}$$

8. $f(x) = \frac{1}{2}x + 3$

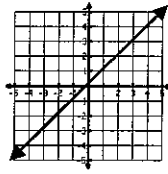
Find x if $f(x) = 11$

$$\begin{aligned} 11 &= \frac{1}{2}x + 3 \\ -3 & \quad -3 \\ 8 &= \frac{1}{2}x \\ \left(\frac{2}{1}\right) 8 &= \frac{1}{2}x \left(\frac{2}{1}\right) \\ 16 &= x \end{aligned}$$

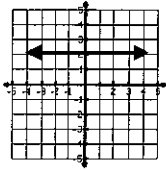
NOTES – Linear Equations

Linear Parent Function

Linear Parent Function – the equation that all other linear equations are based upon ($y = x$)



Horizontal and Vertical Lines (HOYY – VUXX)

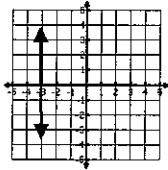


H – horizontal line
 O – “0” slope; $m = \frac{0}{\#} = 0$
 Y – crosses the y-axis
 Y – $y = \#$ (equation)

V – vertical line

U – undefined slope; $m = \frac{\#}{0} = \text{undefined}$

X – crosses the x-axis
 X – $x = \#$ (equation)



Slope

Slope: is the rate of change; $\frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$
 The variable, m , is used for slope.

Slope Formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$
 (x_1, y_1) – one point
 (x_2, y_2) – another point

How to Find the Slope From...

Two Points: Label one point as (x_1, y_1) and another point as (x_2, y_2) . Use the slope formula.

Examples:

* Find the slope between the points $(2, 3)$ and $(5, 9)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 3}{5 - 2} = \frac{6}{3} = 2$$

The slope is 2.

* Find the slope between the points $(-2, 4)$ and $(3, 4)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 4}{3 - (-2)} = \frac{0}{5} = 0$$

The slope is 0. This means it is a horizontal line.

* Find the slope between the points $(-2, 8)$ and $(-2, -2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 8}{-2 - (-2)} = \frac{-10}{0} = \text{undefined}$$

The slope is undefined. This means it is a vertical line.

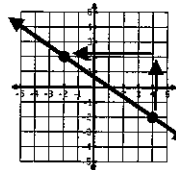
Table: Label one point as (x_1, y_1) and another point as (x_2, y_2) . Use the slope formula.

	x	y	
x_1	-2	4	y_1
x_2	1	-2	y_2
	7	-14	
	10	-20	

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 4}{1 - (-2)} = \frac{-6}{3} = -2$$

The slope is -2.

Graph: Draw two points on the line. Count $\frac{\text{rise}}{\text{run}}$. Note the direction for the sign! Reduce the fraction.



$$m = \frac{\text{rise}}{\text{run}} = \frac{4}{-6} = -\frac{2}{3}$$

The slope is $-\frac{2}{3}$.

Equation: Solve for y . The slope is the number in front of the x . Don't forget the sign.

Examples:

* What is the slope of $y = -\frac{1}{2}x + 5$?
 $m = -\frac{1}{2}$

* What is the slope of $-4x + 3y = 6$

$$\begin{aligned} -4x + 3y &= 6 \\ +4x \quad \quad +4x & \\ \hline 3y &= 4x + 6 \\ \frac{3y}{3} &= \frac{4x}{3} + \frac{6}{3} \\ y &= \frac{4}{3}x + 2 \end{aligned}$$

$$m = \frac{4}{3}$$

* What is the slope of $y = 7$?

$m = 0$ (The equation can be rewritten as $y = 0x + 7$. It graphs a horizontal line.)

* What is the slope of $x = 5$?

The slope is undefined. (The equation cannot be rewritten in $y =$ form. It graphs a vertical line.)

GRAPHING LINEAR EQUATIONS

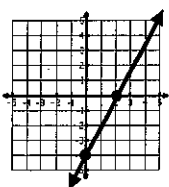
Graphing a linear equation using the intercepts:

1. Find the x-intercept by putting 0 in for y and solving for x.
2. Find the y-intercept by putting 0 in for x and solving for y.
3. Graph the two points on the correct axes and draw a line through the points. Make sure to draw the line all the way across the graph and put arrows on the ends of the line to show that the line continues forever in both directions.

EXAMPLE: Graph $y - 2x = -4$ using the intercepts.

$$\begin{array}{rcl} y - 2x & = & -4 \\ 0 - 2x & = & -4 \\ -2x & = & -4 \\ -2 & -2 & \\ x & = & 2 \end{array} \qquad \begin{array}{rcl} y - 2x & = & -4 \\ y - 2(0) & = & -4 \\ y - 0 & = & -4 \\ y & = & -4 \end{array}$$

The x-intercept is (2, 0) and the y-intercept is (0, -4)



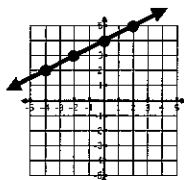
Slope-Intercept Form: $y = mx + b$

m is the slope and b is the y-intercept. To graph an equation from slope-intercept form, identify the slope (m) and y-intercept (b). Make sure to include the correct sign with each number and change the slope to a fraction.

1. Solve for y.
2. Graph the y-intercept, b , on the y-axis.
3. From the y-intercept, use the slope to count $\frac{\text{rise}}{\text{run}}$ to get another point. Watch the sign for the direction.
4. Connect the points with a line. Make sure to put arrows on the ends to show that the line continues forever in both directions.

EXAMPLES:

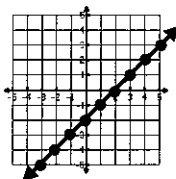
- * Graph $y = -\frac{1}{2}x + 4$
 $m = -\frac{1}{2}$ and $b = 4$



- * Graph $2x - 4 = 2y$.

$$\begin{array}{rcl} 2x - 4 & = & 2y \\ \frac{2x - 4}{2} & = & \frac{2y}{2} \\ x - 2 & = & y \\ y & = & x - 2 \end{array}$$

$m = 1$ and $b = -2$

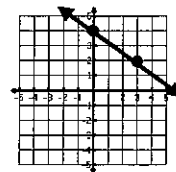


Point-Slope Form: $y - y_1 = m(x - x_1)$

m is the slope, (x_1, y_1) is a specific point on the line, and (x, y) is any point on the line. To graph an equation from point-slope form, distribute and solve for y. The equation is now in slope-intercept form. Graph using the steps for slope-intercept form.

EXAMPLE: Graph $3y - 6 = -2(x - 3)$.

$$\begin{array}{rcl} 3y - 6 & = & -2(x - 3) \\ 3y - 6 & = & -2x + 6 \\ +6 & = & +6 \\ \hline 3y & = & -2x + 12 \\ \frac{3y}{3} & = & \frac{-2x + 12}{3} \\ y & = & -\frac{2}{3}x + 4 \\ m & = & -\frac{2}{3} \text{ and } b = 4 \end{array}$$



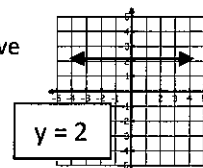
Standard Form: $Ax + By = C$

(x, y) is a point on the line. You have two options to graph an equation that is in standard form.

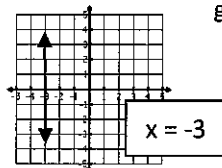
- Option 1:** Solve for y and graph like you would for slope-intercept form.
- Option 2:** Graph like you would using the intercepts.

GRAPHING HORIZONTAL AND VERTICAL LINES

Horizontal Lines: Equations that only have the y variable, graph horizontal lines.



Vertical Lines: Equations that only have the x variable, graph vertical lines.



WRITING EQUATIONS OF LINES

Writing Equations of Lines Given the Slope and the y-intercept:

Substitute the slope in for m and the y-intercept in for b into the slope-intercept form: $y = mx + b$.

Special Cases: If the slope is zero, it is a horizontal line. The equation is $y =$ (the y-value of the point). If the slope is undefined, it is a vertical line. The equation is $x =$ (the x-value of the point).

EXAMPLES:

- * Write the equation of the line that has a y-intercept of 74 and a slope of -2.8. **EQ: $y = -2.8x + 74$**
- * Write the equation of the line that has a y-intercept of 5 and a zero slope. **EQ: $y = 5$**
- * Write the equation of the line that has an undefined slope and passes through (6, 0). **EQ: $x = 6$**

WRITING EQUATIONS OF LINES – cont.

Writing an Equation of a Line Given the Slope and a Point:

1. Substitute the slope in for m and the point in for x_1 and y_1 into the point-slope form:
 $y - y_1 = m(x - x_1)$. Leave the x and y as variables.
2. Distribute and solve for y .

EXAMPLES:

- * Write the equation of the line that has a slope of 2 that passes through the point $(-1, -5)$.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-5) &= 2(x - (-1)) \\ y + 5 &= 2(x + 1) \\ y + 5 &= 2x + 2 \\ \underline{-5 \qquad -5} & \\ y &= 2x - 3 \end{aligned}$$

- * Write the equation of the line that passes through the point $(7, -5)$ and has a slope of $-\frac{1}{4}$.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-5) &= -\frac{1}{4}(x - 7) \\ y + 5 &= -\frac{1}{4}x + \frac{7}{4} \\ \underline{-5 \qquad -5} & \\ y &= -\frac{1}{4}x - \frac{13}{4} \end{aligned}$$

- * Write the equation of the line that has an undefined slope and passes through the point $(1, 8)$.

$x = 1$ (Lines with undefined slopes are vertical lines. The equation for vertical lines is $x = \#$. The $\#$ is the x -value of the point.)

- * Write the equation of the line that has a zero slope and passes through the point $(9, 4)$.

$y = 4$ (Lines with zero slopes are horizontal lines. The equation for horizontal lines is $y = \#$. The $\#$ is the y -value of the point.)

Writing an Equation of a Line Given Two Points:

1. Find the slope between the points by using the slope

$$\text{formula: } m = \frac{y_2 - y_1}{x_2 - x_1}$$

Remember that if the slope is "0", it is a horizontal line ($y =$ the y -value). If the slope is undefined, it is a vertical line ($x =$ the x -value).

2. Use the slope for m and one of the points for x_1 and y_1 into the point-slope form of an equation:

$$y - y_1 = m(x - x_1). \text{ Leave the } x \text{ and } y \text{ as variables.}$$

3. Distribute and solve for y .

EXAMPLES:

- * Find the equation of the line that passes through the points $(-3, 5)$ and $(-2, 9)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 5}{-2 - (-3)} = \frac{4}{1} = 4$$

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 5 &= 4(x - (-3)) \\ y - 5 &= 4(x + 3) \\ y - 5 &= 4x + 12 \\ \underline{+5 \qquad +5} & \\ y &= 4x + 17 \end{aligned}$$

- * Find the equation of the line that passes through the points $(-2, 4)$ and $(-8, 12)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{12 - 4}{-8 - (-2)} = \frac{8}{-6} = -\frac{4}{3}$$

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 4 &= -\frac{4}{3}(x - (-2)) \\ y - 4 &= -\frac{4}{3}(x + 2) \\ y - 4 &= -\frac{4}{3}x - \frac{8}{3} \\ \underline{+4 \qquad +4} & \\ y &= -\frac{4}{3}x + \frac{4}{3} \end{aligned}$$

- * Find the equation of the line that passes through $(-2, 4)$ and $(8, 4)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 4}{8 - (-2)} = \frac{0}{10} = 0$$

$y = 4$ When the slope is zero, it is a horizontal line. Horizontal line equations are $y = \#$. The $\#$ is the y -coordinate of the points.

- * Find the equation of the line that passes through the points $(-3, 4)$ and $(-3, 0)$.

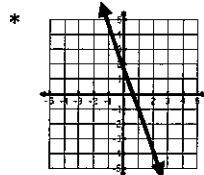
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 4}{-3 - (-3)} = \frac{-4}{0} = \text{undefined}$$

$x = -3$ When the slope is undefined, it is a vertical line. Vertical line equations are $x = \#$. The $\#$ is the x -coordinate of the points.

Writing Equations from a Graph:

If the y-intercept is an integer, find the slope and use the steps for writing an equation given the slope and y-intercept.

EXAMPLE:

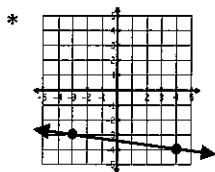


$b = 2$ and $m = -3$

Equation: $y = -3x + 2$

If the y-intercept is not an integer, find the slope and one point that has integers for x and y. Then, follow the steps for writing an equation given the slope and a point.

EXAMPLE:



$m = -\frac{1}{7}$

$$y - y_1 = m(x - x_1)$$

$$y - (-3) = -\frac{1}{7}(x - (-3))$$

$$y + 3 = -\frac{1}{7}(x + 3)$$

$$y + 3 = -\frac{1}{7}x - \frac{3}{7}$$

$$\begin{array}{r} y + 3 = -\frac{1}{7}x - \frac{3}{7} \\ -3 \qquad -3 \\ \hline y = -\frac{1}{7}x - \frac{24}{7} \end{array}$$

PARALLEL AND PERPENDICULAR LINES

Parallel (||) Lines: lines that never intersect. Parallel lines have the same slope.

EXAMPLES:

- * What is the slope of the line parallel to $y = 2.5x - 3$?
|| $m = 2.5$
- * What is the slope of the line parallel to $2x + 3y = 12$?
 $2x + 3y = 12$
 $-2x \quad -2x$
 $3y = 12 - 2x$
 $3 \quad 3 \quad 3$
 $y = 4 - \frac{2}{3}x$ || $m = -\frac{2}{3}$

Perpendicular (⊥) Lines: lines that intersect at a right angle (90°). Perpendicular lines have slopes that are opposite reciprocals (a fraction "flipped" over with the opposite sign).

EXAMPLES:

- * What is the slope of the line perpendicular to $y = -\frac{1}{2}x - 4$?
⊥ $m = 2$
- * What is the slope of the line perpendicular to $x - 3y = 6$?
 $x - 3y = 6$
 $-x \quad -x$
 $-3y = -x + 6$
 $-3 \quad -3 \quad -3$
 $y = \frac{1}{3}x - 2$ ⊥ $m = -3$

Writing Equations for a Parallel Line through a Given Point:

1. Find the slope of the given equation or points.
2. Since parallel lines have the same slope, use that slope for your new equation.
3. Put that slope in for m and the given point in for x_1 and y_1 into the point-slope form of the equation:
 $y - y_1 = m(x - x_1)$.
4. Solve for y .

EXAMPLES:

- * Write the equation of the line parallel to $y = 4x - 3$ that passes through the point $(2, -6)$.
 $x_1 \quad y_1$

|| $m = 4$

$$y - y_1 = m(x - x_1)$$

$$y - (-6) = 4(x - 2)$$

$$y + 6 = 4x - 8$$

$$\begin{array}{r} y + 6 = 4x - 8 \\ -6 \qquad -6 \\ \hline y = 4x - 14 \end{array}$$

- * Write the equation of the line parallel to $x + y = 8$ that passes through the point $(1, 4)$.
 $x_1 \quad y_1$

$$x + y = 8$$

$$\begin{array}{r} -x \quad -x \\ \hline y = -x + 8 \end{array}$$

|| $m = -1$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -1(x - 1)$$

$$y - 4 = -x + 1$$

$$\begin{array}{r} y - 4 = -x + 1 \\ +4 \quad +4 \\ \hline y = -x + 5 \end{array}$$

Writing Equations for a Perpendicular Line through a Given Point:

1. Find the slope of the given equation or points.
2. Since perpendicular lines have opposite reciprocal slopes, make the slope a fraction, flip the fraction over, and change the sign to get the new slope.
3. Put the new slope in for m and the given point in for x_1 and y_1 into the point-slope form of the equation:
 $y - y_1 = m(x - x_1)$.
4. Solve for y .

EXAMPLES:

- * Write the equation of the line perpendicular to $y = -\frac{1}{2}x - 4$ that passes through $(-6, -8)$.
 $x_1 \quad y_1$

⊥ $m = 2$

$$y - y_1 = m(x - x_1)$$

$$y - (-8) = 2(x - (-6))$$

$$y + 8 = 2(x + 6)$$

$$y + 8 = 2x + 12$$

$$\begin{array}{r} y + 8 = 2x + 12 \\ -8 \quad -8 \\ \hline y = 2x + 4 \end{array}$$

- * Write the equation of the line perpendicular to $-6x + 2y = 4$ that passes through $(9, -2)$.
 $x_1 \quad y_1$

$$-6x + 2y = 4$$

$$\begin{array}{r} +6x \quad +6x \\ \hline 2y = 6x + 4 \\ \frac{2y}{2} = \frac{6x}{2} + \frac{4}{2} \\ y = 3x + 2 \end{array}$$

⊥ $m = -\frac{1}{3}$

$$y - y_1 = m(x - x_1)$$

$$y - (-2) = -\frac{1}{3}(x - 9)$$

$$y + 2 = -\frac{1}{3}x + 3$$

$$\begin{array}{r} y + 2 = -\frac{1}{3}x + 3 \\ -2 \qquad -2 \\ \hline y = -\frac{1}{3}x + 1 \end{array}$$

3-6 Writing Linear Functions TEKS 2C

Vocab

Slope-Intercept Form: $y = mx + b$

M= slope: look for words such as per, each, rate, change

B= y-intercept: look for words such as initial, starting,

Examples

It has been observed that a particular plant's growth is directly proportional to time. It measured 2 cm when it arrived at the nursery and 2.5 cm exactly one week later.

a.) Write the function that represents this situation.

$$y = 0.5x + 2$$

b.) Determine the plant's height after 6 weeks.

5

A car rental charge is \$100 per day plus \$0.30 per mile travelled.

a.) Determine a function to represent the DAILY cost to rent the car.

$$y = 0.3x + 100$$

b.) How many miles were travelled in one day if it cost a total of \$190?

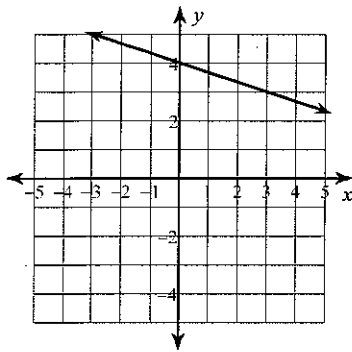
300

Summary:

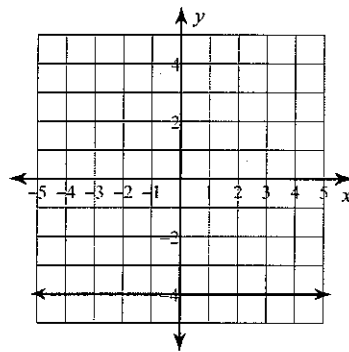
Linear Functions

Write the slope-intercept form of the equation of each line.

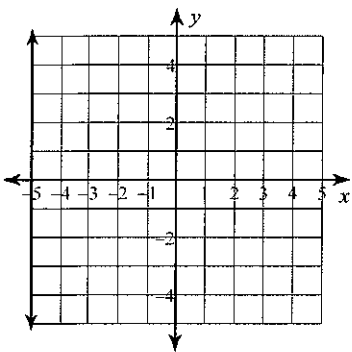
1)



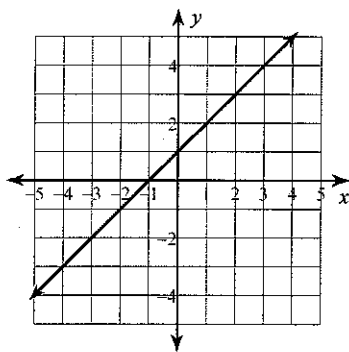
2)



3)



4)



Write the slope-intercept form of the equation of each line given the slope and y-intercept.

5) Slope = $-\frac{1}{2}$, y-intercept = 1

6) Slope = 0, y-intercept = 1

Write the slope-intercept form of the equation of each line.

7) $x + y = 5$

8) $10x - 7y = -56$

9) $y = -\frac{4}{5}(x + 5)$

10) $y + 4 = -2(x - 2)$

Write the slope-intercept form of the equation of the line through the given point with the given slope.

11) through: $(3, 4)$, slope = -1

12) through: $(5, 3)$, slope = undefined

13) through: $(4, -5)$, slope = -8

14) through: $(5, 4)$, slope = 0

Write the slope-intercept form of the equation of the line through the given points.

15) through: $(3, 1)$ and $(2, 2)$

16) through: $(0, 3)$ and $(3, -2)$

17) through: $(-5, -4)$ and $(2, -4)$

18) through: $(-3, -4)$ and $(-3, -5)$

Write the slope-intercept form of the equation of the line described.

19) through: $(2, 0)$, parallel to $y = -\frac{1}{2}x - 4$

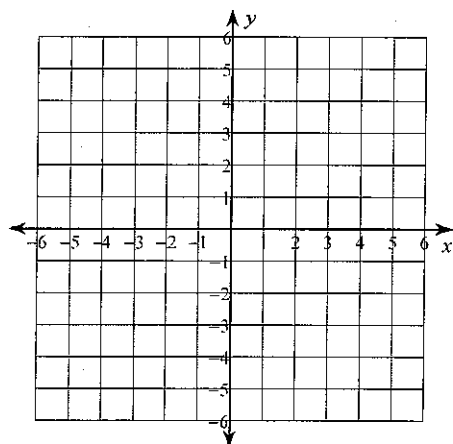
20) through: $(-3, 3)$, parallel to $y = -2x - 4$

21) through: $(5, -4)$, perp. to $x = 0$

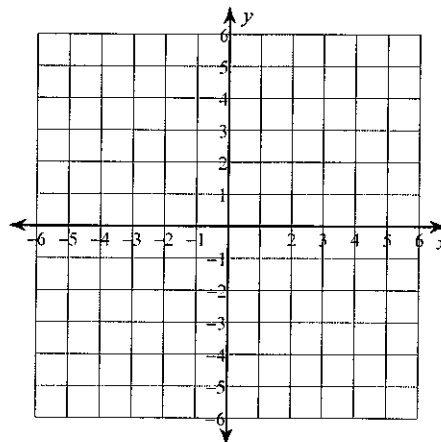
22) through: $(2, -4)$, perp. to $y = -4$

Sketch the graph of each line.

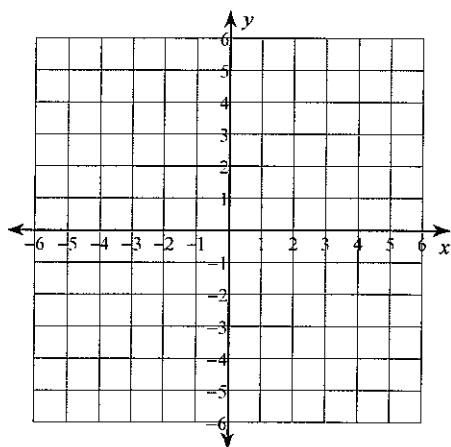
23) $y = -\frac{1}{5}x + 5$



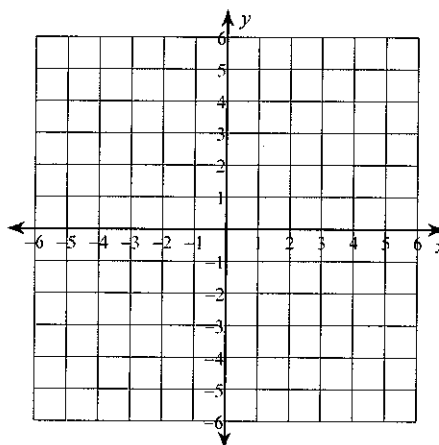
24) $x = 3$



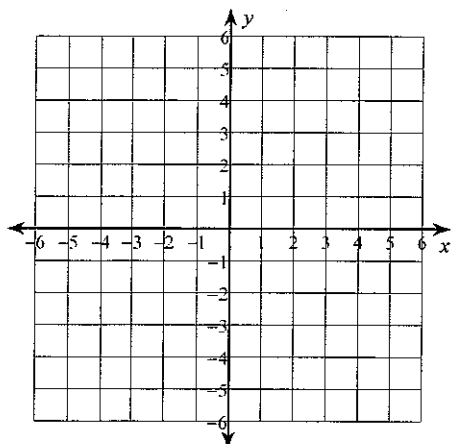
25) $y = -4$



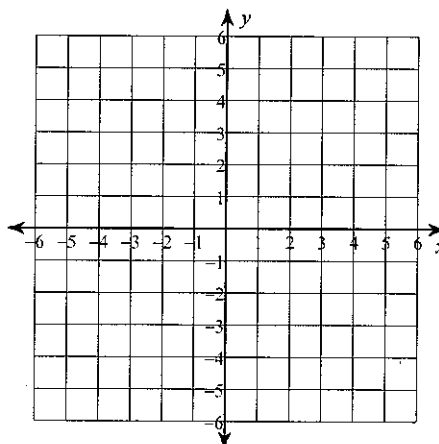
26) $2x - y = -1$



27) $2x - y = 2$



28) $8x + 3y = -9$



Answers to Linear Functions

1) $y = -\frac{1}{3}x + 4$

2) $y = -4$

3) $x = -5$

4) $y = x + 1$

5) $y = -\frac{1}{2}x + 1$

6) $y = 1$

7) $y = -x + 5$

8) $y = \frac{10}{7}x + 8$

9) $y = -\frac{4}{5}x - 4$

10) $y = -2x$

11) $y = -x + 7$

12) $x = 5$

13) $y = -8x + 27$

14) $y = 4$

15) $y = -x + 4$

16) $y = -\frac{5}{3}x + 3$

17) $y = -4$

18) $x = -3$

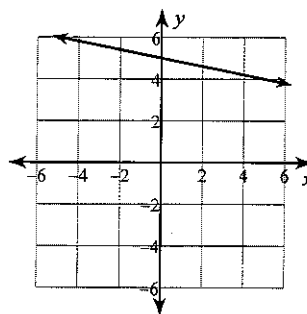
19) $y = -\frac{1}{2}x + 1$

20) $y = -2x - 3$

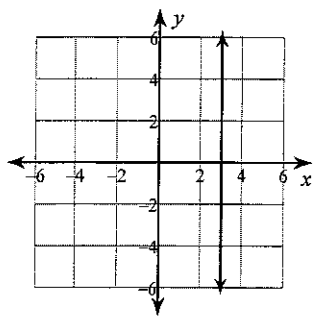
21) $y = -4$

22) $x = 2$

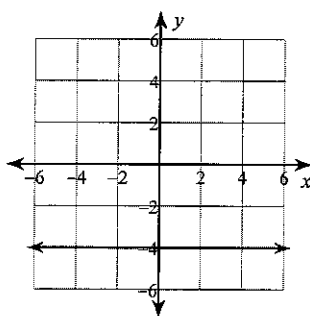
23)



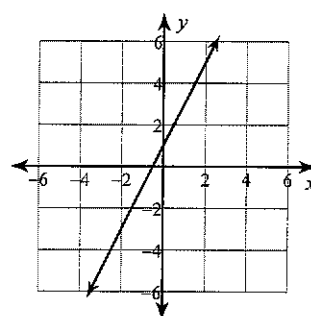
24)



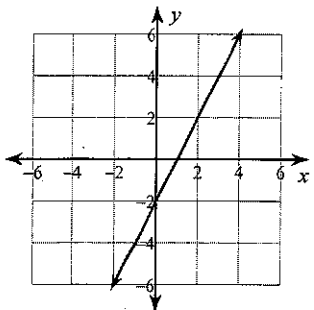
25)



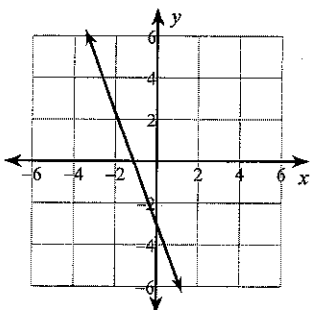
26)



27)



28)



INTRODUCTION TO SYSTEMS OF LINEAR EQUATIONS

System of Linear Equations

Solution

Two or more linear equations with the same variables.

An ordered pair that is true for ALL of the linear equations in the system.
On a graph, a solution is the point of intersection.

EX:

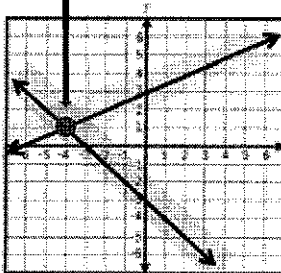
$$y = \frac{1}{2}x + 3$$

$$y = -x - 3$$

x	y
-6	0
-4	1
0	3
1	3.5

x	y
-6	3
-4	1
0	-3
1	-4

Solution: $(-4, 1)$



$$y = \frac{1}{2}x + 3$$

$$y = -x - 3$$

$$1 = \frac{1}{2}(-4) + 3$$

$$1 = -(-4) - 3$$

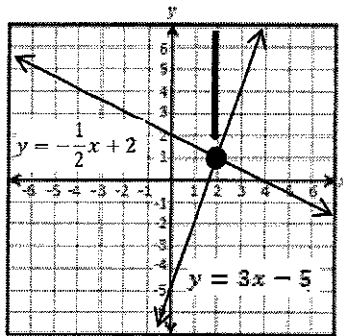
$$1 = -2 + 3$$

$$1 = 4 - 3$$

$$1 = 1$$

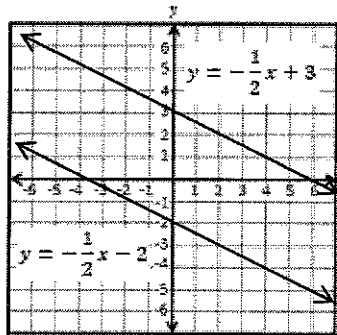
$$1 = 1$$

SOLUTIONS TO SYSTEMS OF LINEAR EQUATIONS



1 SOLUTION
 $(2, 1)$

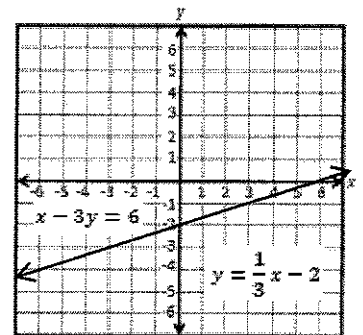
Point of Intersection



NO SOLUTIONS

Parallel Lines never intersect

Same Slope
Different y-intercepts



INFINITELY MANY SOLUTIONS

Same line, every point on the line is a solution

Same Slope
Same y-intercepts

SOLVING SYSTEMS OF LINEAR EQUATIONS

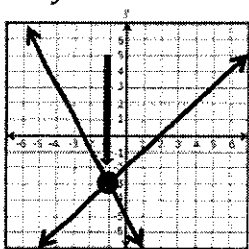
GRAPHING

Graph all of the equations on a coordinate plane.

Find the point, or points of intersection.

$$y = x - 2$$

$$y = -2x - 5$$



Solution: $(-1, -3)$

SUBSTITUTION

Use algebraic methods to solve for the solution.

Solve one equation for a variable. Then, replace the variable in the second equation with its equivalent from the first equation.

$$y = x - 2$$

$$2x + y = -5$$

$$2x + (x - 2) = -5 \quad y = x - 2$$

$$3x - 2 = -5 \quad y = (-1) - 2$$

$$3x = -3 \quad y = -3$$

$$x = -1$$

Solution: $(-1, -3)$

ELIMINATION

Add or subtract equations to eliminate a variable.

$$x - y = 2$$

$$2x + y = -5$$

$$\begin{array}{r} x - y = 2 \\ 2x + y = -5 \\ \hline 3x = -3 \end{array} \quad \begin{array}{r} x - y = 2 \\ (-1) - y = 2 \\ \hline -y = 3 \\ y = -3 \end{array}$$

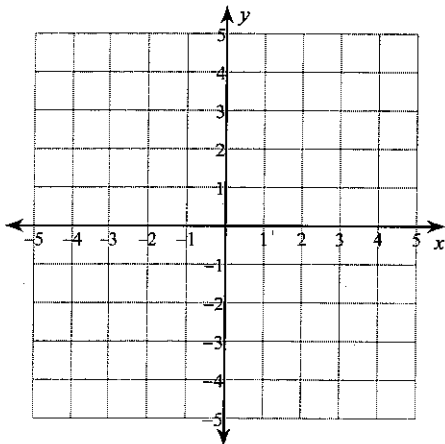
Solution: $(-1, -3)$

Systems of Linear Equations Practice

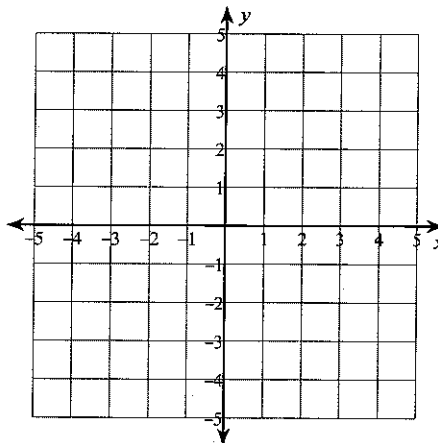
Solve each system by graphing.

1) $y = -\frac{1}{2}x + 3$

$$y = -\frac{3}{2}x + 1$$



2) $y = -x - 4$
 $y = -x + 2$



Solve each system by substitution.

3) $y = 3x + 11$

$$y = 8x + 21$$

4) $2x - 4y = -20$

$$y = 5$$

5) $-8x + y = 7$

$$24x - 3y = -21$$

6) $-x - 3y = -6$

$$2x + 6y = -7$$

Solve each system by elimination.

7) $6x - 2y = -16$

$$10x + 2y = 16$$

8) $2x + 6y = -21$

$$2x + 6y = -30$$

9) $8x + 2y = -6$

$$16x - 9y = -25$$

10) $24x + 30y = 30$

$$16x + 20y = 20$$

Answers to Systems of Linear Equations Practice

- 1) $(-2, 4)$ 2) No solution 3) $(-2, 5)$ 4) $(0, 5)$
5) Infinite number of solutions 6) No solution 7) $(0, 8)$
8) No solution 9) $(-1, 1)$ 10) Infinite number of solutions

EXPONENT RULES & PRACTICE

1. **PRODUCT RULE:** To multiply when two bases are the same, write the base and ADD the exponents.

$$x^m \cdot x^n = x^{m+n}$$

Examples:

A. $x^3 \cdot x^8 = x^{11}$

B. $2^4 \cdot 2^2 = 2^6$

C. $(x^2y)(x^3y^4) = x^5y^5$

2. **QUOTIENT RULE:** To divide when two bases are the same, write the base and SUBTRACT the exponents.

$$\frac{x^m}{x^n} = x^{m-n}$$

Examples:

A. $\frac{x^5}{x^2} = x^3$

B. $\frac{3^5}{3^3} = 3^2$

C. $\frac{x^2y^5}{xy^3} = xy^2$

3. **ZERO EXPONENT RULE:** Any base (except 0) raised to the zero power is equal to one.

$$x^0 = 1$$

Examples:

A. $y^0 = 1$

B. $6^0 = 1$

C. $(7a^3b^{-1})^0 = 1$

4. **POWER RULE:** To raise a power to another power, write the base and MULTIPLY the exponents.

$$(x^m)^n = x^{m \cdot n}$$

Examples:

A. $(x^3)^2 = x^6$

B. $(3^2)^4 = 3^8$

C. $(z^5)^2 = z^{10}$

5. **EXPANDED POWER RULE:**

$$(xy)^m = x^m y^m \quad \left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$$

Examples:

A. $(2a)^3 = 2^3 a^3 = 8a^3$

C. $\left(\frac{x^2}{y}\right)^4 = \frac{(x^2)^4}{y^4} = \frac{x^8}{y^4}$

B. $(6x^3)^2 = 6^2 (x^3)^2 = 36x^6$

D. $\left(\frac{2x}{3y^2}\right)^3 = \frac{(2x)^3}{(3y^2)^3} = \frac{2^3 x^3}{3^3 (y^2)^3} = \frac{8x^3}{27y^6}$

6. **NEGATIVE EXPONENTS:** If a factor in the numerator or denominator is moved across the fraction bar, the sign of the exponent is changed.

$$x^{-m} = \frac{1}{x^m} \quad \frac{1}{x^{-m}} = x^m \quad \left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n$$

Examples:

A. $x^{-3} = \frac{1}{x^3}$

B. $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

C. $-4x^5y^{-2} = \frac{-4x^5}{y^2}$

D. $\left(\frac{x^2}{y}\right)^{-3} = \left(\frac{y}{x^2}\right)^3 = \frac{y^3}{x^6}$

E. $(3x^{-2}y)(-2xy^{-3}) = -6x^{-1}y^{-2} = \frac{-6}{xy^2}$

F. $\frac{a^{-2}b^3}{c^{-4}d^{-1}} = \frac{b^3c^4d}{a^2}$

G. $(-2x^2y^{-4})^{-2} = \left(\frac{-2x^2}{y^4}\right)^{-2} = \left(\frac{y^4}{-2x^2}\right)^2 = \frac{y^8}{4x^4}$

CAUTION: $-x \neq \frac{1}{x}$ For example: $-3 \neq \frac{1}{3}$

REMEMBER: An exponent applies to only the factor it is directly next to *unless* parentheses enclose other factors.

Examples:

A. $(-3)^2 = (-3)(-3) = 9$

B. $-3^2 = -9$

LAWS OF EXPONENTS PRACTICE

1. FIND THE VALUE OF EACH EXPRESSION:

a) $(-6)^3 =$

b) $9^0 - 3 =$

c) $100^2 =$

d) $\left(\frac{3}{5}\right)^2 =$

e) $1^7 =$

f) $3^{-3} =$

g) $-11^2 =$

h) $(-11)^2 =$

i) $\left(\frac{3}{5}\right)^{-2} =$

2. SIMPLIFY EACH PRODUCT:

a) $10^{12} \cdot 10^{35} =$

b) $a^{-7} \cdot a^{12} =$

c) $c^3 \cdot c^8 =$

d) $d^7 \cdot d^{-9} =$

e) $a^6 \cdot b^5 =$

f) $4^{-4} \cdot 4^2 =$

g) $(2x^2)(4x^{-3}y^2) =$

h) $(-3a^2b)(6ab^4c) =$

i) $(7q^{-5})(12q^3r^5) =$

j) $(11c^8)(-10c^4d) =$

k) $(9x^{10}z^2)(-x^5y^3) =$

l) $(-8f^6g)(-7f^2g^5h) =$

3. SIMPLIFY EACH EXPRESSION:

a) $(x^2)^3 =$

b) $(5^2)^3 =$

c) $(y^5)^{-4} =$

d) $(4y^3)^2 =$

e) $(8c^5)^{-2} =$

f) $(-3h^9)^3 =$

g) $(y^{-4}d^6)^8 =$

h) $(-5h^9k^{-7})^3 =$

i) $(k^9)^5(k^3)^2 =$

j) $(3y^6)^2(x^5y^2z) =$

4) Write the expression in expanded form: $3a^5b^3$?

5) Write the expression in exponential form: $8 \cdot x \cdot x \cdot x \cdot x \cdot y \cdot z \cdot z$

6. SIMPLIFY EACH QUOTIENT AND THEN FIND THE VALUE OF THE RESULT:

a) $\frac{10^6}{10^2} =$

b) $\frac{4^{17}}{4^{14}} =$

c) $\frac{9^{210}}{9^{207}} =$

d) $\frac{2y^2}{2y^2} =$

e) $\frac{8^4}{8^1} =$

7. SIMPLIFY EACH EXPRESSION:

a) $\left(\frac{5c}{d^2}\right)^2 =$

b) $\left(\frac{4d^3}{c^5}\right)^{-3} =$

c) $\left(\frac{2d^4}{4e}\right)^3 =$

d) $\frac{6r^3}{2r} =$

e) $\frac{-40s^6}{20s^9} =$

f) $\frac{21d^{18}e^5}{7d^{11}e^{-3}} =$

g) $\frac{-16w^7r^2}{-4wr} =$

Laws of Exponents Answer Key

Name: ANSWER KEY

1. FIND THE VALUE OF EACH EXPRESSION:

a) $(-6)^2 = -216$ b) $9^0 - 3 = 1 - 3 = -2$ c) $100^2 = 10000$ d) $(\frac{3}{5})^2 = \frac{9}{25}$

e) $1^2 = 1$ f) $3^{-3} = \frac{1}{3^3} = \frac{1}{27}$ g) $-11^2 = -121$ h) $(-11)^2 = 121$ i) $(\frac{3}{5})^{-2} = (\frac{5}{3})^2 = \frac{25}{9}$

2. SIMPLIFY EACH PRODUCT:

a) $10^{12} \cdot 10^{35} = 10^{47}$ b) $a^{-7} \cdot a^{12} = a^5$ c) $c^3 \cdot c^8 = c^{11}$

d) $d^7 \cdot d^{-9} = d^{-2} = \frac{1}{d^2}$ e) $a^6 \cdot b^5 = a^6 b^5$ f) $4^{-4} \cdot 4^2 = 4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

g) $(2x^2)(4x^{-3}y^2) = 8x^{-1}y^2 = \frac{8y^2}{x}$ h) $(-3a^2b)(6ab^4c) = -18a^3b^5c$ i) $(7q^{-3})(12q^4r^5) = 84q^{-2}r^5 = \frac{84r^5}{q^2}$

j) $(11c^3)(-10c^3d) = -110c^{12}d$ k) $(9x^{10}z^2)(-x^5y^3) = -9x^{15}z^2y^3$ l) $(-8f^6g)(-7f^2g^4h) = 56f^8g^4h$

3. SIMPLIFY EACH EXPRESSION:

a) $(x^2)^3 = x^6$

b) $(5^2)^3 = 5^6$

c) $(y^5)^{-2} = y^{-10} = \frac{1}{y^{10}}$

d) $(4y^3)^2 = 4^2(y^3)^2 = 16y^6$

e) $(8c^2)^{-2} = 8^{-2}c^{-4} = \frac{1}{64c^4}$

f) $(-3h^3)^3 = (-3)^3(h^3)^3 = -27h^9$

g) $(y^{-4}d^6)^3 = y^{-12}d^{18} = \frac{d^{18}}{y^{12}}$

h) $(-5h^2k^{-1})^3 = (-5)^3(h^2)^3(k^{-1})^3 = -125h^6k^{-3} = \frac{-125h^6}{k^3}$

i) $(k^9)(k^2)^3 = k^9 \cdot k^6 = k^{15}$

j) $(3y^6)^2(x^5y^2z) = 3^2(y^6)^2(x^5y^2z) = 9y^{12}x^5y^2z = 9y^{14}x^5z$

4) Write the expression in expanded form: $3a^3b^3$? $3 \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b$

5) Write the expression in exponential form: $8 \cdot x \cdot x \cdot x \cdot x \cdot y \cdot z \cdot z$ $8x^4yz^2$

6. SIMPLIFY EACH QUOTIENT AND THEN FIND THE VALUE OF THE RESULT:

a) $\frac{10^6}{10^2} = 10^4$

b) $\frac{4^{17}}{4^{14}} = 4^3 = 64$

c) $\frac{9^{210}}{9^{207}} = 9^3 = 729$

d) $\frac{2y^2}{2y^2} = 1$

e) $\frac{8^4}{8^1} = 8^3 = 512$

7. SIMPLIFY EACH EXPRESSION:

$$a) \left(\frac{5c}{d^2}\right)^2 = \frac{5^2 c^2}{d^4}$$

$$\boxed{\frac{25c^2}{d^4}}$$

$$b) \left(\frac{4d^3}{c^5}\right)^{-3} = \frac{4^{-3} d^{-9}}{c^{-15}}$$

$$= \frac{c^{15}}{4^3 d^9} = \boxed{\frac{c^{15}}{64d^9}}$$

$$c) \left(\frac{2d^4}{4e}\right)^3 = \frac{2^3 d^{12}}{4^3 e^3} = \frac{8d^{12}}{64e^3} = \boxed{\frac{1d^{12}}{8e^3}}$$

$$d) \frac{6r^3}{2r} = \boxed{3r^2}$$

$$e) \frac{-40s^6}{20s^9} = \boxed{\frac{-2}{s^3}}$$

$$f) \frac{21d^{18}e^5}{7d^{11}e^{-3}} = \frac{3d^7e^8}{1} = \boxed{3d^7e^8}$$

$$g) \frac{-16w^2r^2}{-4wr} = \frac{4w^1r^1}{1} = \boxed{4w^1r^1}$$