Foundations of Algebra Teaching & Learning Framework

Semester 1-Block Schedule				
Module 1 2 weeks	**Module 2** 3 weeks	**Module 3** 3 weeks	**Module 4** 5 weeks	**Module 5** 5 weeks
Number Sense & Quantity	**Arithmetic to Algebra**	**Proportional Reasoning**	**Equations & Inequalities**	**Quantitative Reasoning with Functions**
MFANSQ1 Relationships (4 operations; multiples of fractions; multiple/divide by powers of ten with decimals; compare fractions/decimals) **MFANSQ2 +/-** (Real number+- on a number line & the meaning of zero) **MFANSQ3 Irrationals** (Irrational number approximations; adding & multiplying with rational & irrational numbers) **MFANSQ4 Computation** (Compute multi-digit decimals; compute with rational numbers; division of fractions by fractions; multi-step problems with any form of rational number)	**MFAAA1 Equivalent Expressions** (Commutative & distributive properties; numerical & algebraic expressions; add, subtract & multiply algebraic expressions; equivalent expressions; evaluate formulas) **MFAAA2 Exponents** (integer exponents; formulas; square & cube roots; Pythagorean Theorem) **Algebra Unit 3:** **Investigating Rational and Irrational Numbers** **A.NR.5** **A.NR.5.1** (Simplify radicals) **A.NR.5.2** (Explain irrational sums and products)	**MFAPR1 Equivalent Ratios** (Equivalent ratios) **MFAPR2 Proportions** (Fraction equivalence & division; percent problems) **MFAPR3 Graphing** (Unit rates as slope; similar triangles and slope; compare proportions in multiple representations)	**MFAEI1 One Variable** (Solve equations & inequalities & justify solutions) **MFAEI2 Units** (Scale, units, graphing) **MFAEI3 Two Variables** (Algebraic models; graphing calculators; systems of equations) **MFAEI4 Literal Equations** (Solve for a specific variable) **Algebra Unit 2:** **Analyzing Linear Inequalities** **A.PAR.4** **A.PAR.4.1** (Create, solve, and graph linear inequalities) **A.PAR.4.2** (Constraints of linear inequalities) **A.PAR.4.3** (Systems of linear inequalities)	**MFAQR1 Characteristics** (Domain & range) **MFAQR2 Compare & Graph** (Rates of change; linear & non-linear; key features; compare with multiple representations) **MFAQR3 Construct & Interpret** (Write; variables in context; function notation) **Algebra Unit 1:** **Modeling Linear Functions** **A.FGR.2** **A.FGR.2.1** (Arithmetic sequences) **A.FGR.2.2** (Construct and interpret linear functions) **A.FGR.2.3** (Domain and range) **A.FGR.2.4** (Function notation) **A.FGR.2.5** (Analyze linear and non-linear)
These units were written to build upon concepts from prior units, so later units contain tasks that depend upon the concepts addressed in earlier units. All units will include the Mathematical Practices and indicate skills to maintain				

NOTE: Mathematical standards are interwoven and should be addressed throughout the year in as many different units and tasks as possible in order to stress the natural connections that exist among mathematical topics
NSQ- number sense & quantity; **AA-** arithmetic to algebra; **PR-** proportional reasoning; **EI-** equations and inequalities; **QR-** quantitative reasoning with functions

Cobb County School District
2023-2024

MATHEMATICS

Algebra Teaching & Learning Framework Block

Semester 1			Semester 2					
Foundation of Algebra Content Units 1-3			**Unit 4** 6 weeks	**Unit 5** 2 weeks	**Unit 6** 4 weeks	**Unit 7** 3 weeks	**Unit 8** 2 weeks	**Unit 9** 1 week
Modeling Linear Functions A.FGR.2	**Analyzing Linear Inequalities A.PAR.4**	**Investigating Rational and Irrational Numbers A.NR.5**	**Modeling and Analyzing Quadratic Functions A.PAR.6 A.FGR.7**	**Modeling and Analyzing Exponential Expressions & Equations A.PAR.8**	**Analyzing Exponential Functions A.FGR.9**	**Investigating Data A.DSR.10**	**Algebraic Connections to Geometric Concepts A.GSR.3**	**Culminating Capstone Unit**
A.FGR.2.1 (Arithmetic sequences) **A.FGR.2.2** (Construct and interpret linear functions) **A.FGR.2.3** (Domain and range) **A.FGR.2.4** (Function notation) **A.FGR.2.5** (Analyze linear and non-linear)	**A.PAR.4.1** (Create, solve, and graph linear inequalities) **A.PAR.4.2** (Constraints of linear inequalities) **A.PAR.4.3** (Systems of linear inequalities)	**A.NR.5.1** (Simplify radicals) **A.NR.5.2** (Explain irrational sums and products)	**A.PAR.6.1** (Interpret quadratic expressions) **A.PAR.6.2** (Rewrite quadratic expressions) **A.PAR.6.3** (Create and solve quadratic equations) **A.PAR.6.4** (Constraints of quadratic equations) **A.FGR.7.1** (Build and evaluate functions) **A.FGR.7.2** (Transformations) **A.FGR.7.3** (Analyze characteristics of quadratic functions) **A.FGR.7.4** (Domain and range) **A.FGR.7.5** (Rewrite quadratic functions to find max/min) **A.FGR.7.6** (Create and graph quadratic functions) **A.FGR.7.7** (Average rate of change) **A.FGR.7.8** (Write a quadratic function for different properties) **A.FGR.7.9** (Compare functions represented differently)	**A.PAR.8.1** (Interpret exponential expressions) **A.PAR.8.2** (Create exponential equations in one variable) **A.PAR.8.3** (Create exponential equations in two variables) **A.PAR.8.4** (Constraints of exponential equations)	**A.FGR.9.1** (Build and evaluate functions) **A.FGR.9.2** (Graph and analyze characteristics of exponential functions) **A.FGR.9.3** (Transformations) **A.FGR.9.4** (Geometric sequences) **A.FGR.9.5** (Compare functions represented differently)	**A.DSR.10.1** (Compare center and variability with appropriate statistics) **A.DSR.10.2** (Interpret shape, center, and variability) **A.DSR.10.3** (Represent data on a scatter plot) **A.DSR.10.4** (Interpret slope and y-intercept of linear model) **A.DSR.10.5** (Line of best fit and r) **A.DSR.10.6** (Choose appropriate function from data) **A.DSR.10.7** (Correlation vs. Causation)	**A.GSR.3.1** (Solve problems with slope, parallel and perpendicular lines, area, and perimeter) **A.GSR.3.2** (Apply distance formula, midpoint formula, and slope to solve problems)	**All standards**

Units contain tasks that depend upon the concepts addressed in earlier units. Mathematical standards are interwoven and should be addressed throughout the year in as many different units and tasks as possible in order to stress the natural connections that exist among mathematical topics.

The Framework for Statistical Reasoning, Mathematical Modeling Framework, and the K-12 Mathematical Practices should be taught throughout the units.

Key for Course Standards: MP: Mathematical Practices, MM: Mathematical Modeling, NR: Numerical Reasoning, FGR: Functional & Graphical Reasoning, AGR: Algebraic & Geometric Reasoning, GSR: Geometric & Spatial Reasoning, PAR: Patterning & Algebraic Reasoning, DSR: Data & Statistical Reasoning

Updated 3/15/2023

Richard Woods, Georgia's School Superintendent
"Educating Georgia's Future"

Standards in Mathematics
Foundations of Algebra

K-12 Mathematics Introduction

Georgia Mathematics focuses on actively engaging the student in the development of mathematical understanding by working independently and cooperatively to solve problems, estimating and computing efficiently, using appropriate tools, concrete models and a variety of representations, and conducting investigations and recording findings. There is a shift toward applying mathematical concepts and skills in the context of authentic problems and student understanding of concepts rather than merely following a sequence of procedures. In mathematics classrooms, students will learn to think critically in a mathematical way with an understanding that there are many different solution pathways and sometimes more than one right answer in applied mathematics. Mathematics is the economy of information. The central idea of all mathematics is to discover how knowing some things leads, via reasoning, to knowing more—without having to commit the information to memory as a separate fact. It is the reasoned, logical connections that make mathematics manageable. The implementation of Georgia Performance Standards in Mathematics places the expected emphasis on sense-making, problem solving, reasoning, representation, modeling, representation, connections, and communication.

Foundations of Algebra

Foundations of Algebra is a first year high school mathematics course option for students who have completed mathematics in grades 6 – 8 yet will need substantial support to bolster success in high school mathematics. The course is aimed at students who have reported low standardized test performance in prior grades and/or have demonstrated significant difficulties in previous mathematics classes.

Foundations of Algebra will provide many opportunities to revisit and expand the understanding of foundational algebra concepts, will employ diagnostic means to offer focused interventions, and will incorporate varied instructional strategies to prepare students for required high school mathematics courses. The course will emphasize both algebra and numeracy in a variety of contexts including number sense, proportional reasoning, quantitative reasoning with functions, and solving equations and inequalities.

Instruction and assessment should include the appropriate use of manipulatives and technology. Mathematics concepts should be represented in multiple ways, such as concrete/pictorial, verbal/written, numeric/data-based, graphical, and symbolic. Concepts should be introduced and used, where appropriate, in the context of realistic experiences.

Standards in Mathematics
Foundations of Algebra

The Standards for Mathematical Practice will provide the foundation for instruction and assessment. The content standards are an amalgamation of mathematical standards addressed in grades 3 through high school. The standards from which the course standards are drawn are identified for reference.

Mathematics | Standards for Mathematical Practice

Mathematical Practices are listed with each grade's mathematical content standards to reflect the need to connect the mathematical practices to mathematical content in instruction.

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1 Make sense of problems and persevere in solving them.
High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Standards in Mathematics
Foundations of Algebra

2 Reason abstractly and quantitatively.
High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.
High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.
High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.
High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools

might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision. High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure. By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.

8 Look for and express regularity in repeated reasoning.
High school students notice if calculations are repeated, and look both for general methods and for shortcuts.

Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics should engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. **Students who do not have an understanding of a topic may rely on procedures too heavily.** Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. **In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.**

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Foundations of Algebra | Content Standards

Number Sense and Quantity

Students will compare different representations of numbers (i.e. fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ1. Students will analyze number relationships.

a. Solve multi-step real world problems, analyzing the relationships between all four operations. *For example, understand division as an unknown-factor problem in order to solve problems. Knowing that 50 x 40 = 2000 helps students determine how many boxes of cupcakes they will need in order to ship 2000 cupcakes in boxes that hold 40 cupcakes each.* (MCC3.OA.6, MCC4.OA.3)
b. Understand a fraction a/b as a multiple of 1/b. (MCC4.NF.4)
c. Explain patterns in the placement of decimal points when multiplying or dividing by powers of ten. (MCC5.NBT.2)
d. Compare fractions and decimals to the thousandths place. *For fractions, use strategies other than cross multiplication. For example, locating the fractions on a number line or using*

benchmark fractions to reason about relative size. For decimals, use place value. (MCC4.NF.2;MCC5.NBT.3,4)

MFANSQ2. Students will conceptualize positive and negative numbers (including decimals and fractions).

a. Explain the meaning of zero. (MCC6.NS.5)
b. Represent numbers on a number line. (MCC6.NS.5,6)
c. Explain meanings of real numbers in a real world context. (MCC6.NS.5)

MFANSQ3. Students will recognize that there are numbers that are not rational, and approximate them with rational numbers.

a. Find an estimated decimal expansion of an irrational number locating the approximations on a number line. *For example, for $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue this pattern in order to obtain better approximations.* (MCC8.NS.1,2)
b. Explain the results of adding and multiplying with rational and irrational numbers. (MCC9-12.N.RN.3)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.

a. Find sums, differences, products, and quotients of multi-digit decimals using strategies based on place value, the properties of operations, and/or relationships between operations. (MCC5.NBT.7; MCC6.NS.3)
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MCC7.NS.1,2)
c. Interpret and solve contextual problems involving division of fractions by fractions. *For example, how many 3/4-cup servings are in 2/3 of a cup of yogurt?* (MCC6.NS.1)
d. Illustrate and explain calculations using models and line diagrams. (MCC7.NS.1,2)
e. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using estimation strategies and graphing technology. (MCC7.NS.3, MCC7.E.3, MCC9-12.N.Q.3)

Arithmetic to Algebra

Students will extend arithmetic operations to algebraic modeling.

MFAAA1. Students will generate and interpret equivalent numeric and algebraic expressions.

a. Apply properties of operations emphasizing when the commutative property applies. (MCC7.EE.1)

Standards in Mathematics
Foundations of Algebra

b. Use area models to represent the distributive property and develop understandings of addition and multiplication (all positive rational numbers should be included in the models). (MCC3.MD.7)
c. Model numerical expressions (arrays) leading to the modeling of algebraic expressions. (MCC7.EE.1,2; MCC9-12.A.SSE.1,3)
d. Add, subtract, and multiply algebraic expressions. (MCC6.EE.3, MCC6.EE.4, MC7.EE.1, MCC9-12.A.SSE.3)
e. Generate equivalent expressions using properties of operations and understand various representations within context. *For example, distinguish multiplicative comparison from additive comparison. Students should be able to explain the difference between "3 more" and "3 times".* (MCC4.0A.2; MCC6.EE.3, MCC7.EE.1,2;MCC9-12.A.SSE.3)
f. Evaluate formulas at specific values for variables. *For example, use formulas such as A = l x w and find the area given the values for the length and width*. (MCC6.EE.2)

MFAAA2. Students will interpret and use the properties of exponents.

a. Substitute numeric values into formulas containing exponents, interpreting units consistently. (MCC6.EE.2, MCC9-12.N.Q.1, MCC9-12.A.SSE.1, MCC9-12.N.RN.2)
b. Use properties of integer exponents to find equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$. (MCC8.EE.1)
c. Evaluate square roots of perfect squares and cube roots of perfect cubes (MCC8.EE.2)
d. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. (MCC8.EE.2)
e. Use the Pythagorean Theorem to solve triangles based on real-world contexts (Limit to finding the hypotenuse given two legs). (MCC8.G.7)

Proportional Reasoning

Students will use ratios to solve real-world and mathematical problems.

MFAPR1. Students will explain equivalent ratios by using a variety of models. *For example, tables of values, tape diagrams, bar models, double number line diagrams, and equations*. (MCC6.RP.3)

MFAPR2. Students will recognize and represent proportional relationships between quantities.

a. Relate proportionality to fraction equivalence and division. *For example,* $\frac{3}{6}$ *is equal to* $\frac{4}{8}$ *because both yield a quotient of ½ and, in both cases, the denominator is double the value of the numerator.* (MCC4.NF.1)

Standards in Mathematics
Foundations of Algebra

b. Understand real-world rate/ratio/percent problems by finding the whole given a part and find a part given the whole. (MCC6.RP.1,2,3;MCC7.RP.1,2)
c. Use proportional relationships to solve multistep ratio and percent problems. (MCC7.RP.2,3)

MFAPR3. Students will graph proportional relationships.

a. Interpret unit rates as slopes of graphs. (MCC8.EE.5)
b. Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane. (MCC8.EE.6)
c. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* (MCC8.EE.5)

Equations and Inequalities

Students will solve, interpret, and create linear models using equations and inequalities.

MFAEI1. Students will create and solve equations and inequalities in one variable.

a. Use variables to represent an unknown number in a specified set. (MCC.6.EE2,5,6)
b. Explain each step in solving simple equations and inequalities using the equality properties of numbers. (MCC9-12.A.REI.1)
c. Construct viable arguments to justify the solutions and methods of solving equations and inequalities. (MCC9-12.A.REI.1)
d. Represent and find solutions graphically.
e. Use variables to solve real-world and mathematical problems. (MCC6.EE.7,MCC7.EE.4)

MFAEI2. Students will use units as a way to understand problems and guide the solutions of multi-step problems.

a. Choose and interpret units in formulas. (MCC9-12.N.Q.1)
b. Choose and interpret graphs and data displays, including the scale and comparisons of data. (MCC3.MD.3, MCC9-12.N.Q.1)
c. Graph points in all four quadrants of the coordinate plane. (MCC6.NS.8)

MFAEI3. Students will create algebraic models in two variables.

a. Create an algebraic model from a context using two-variable equations. (MCC6.EE.6,~~8~~; MCC8.EE.8; MCC9-12.A.CED.2)
b. Find approximate solutions using technology to graph, construct tables of values, and find successive approximations. (MCC9-12.A.REI.10,11)

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c. Represent solutions to systems of equations graphically or by using a table of values. (MCC6.EE.5; MCC7.EE3; MCC8.EE.8; CC9-12.A.CED.2)
d. Analyze the reasonableness of the solutions of systems of equations within a given context. (MCC6.EE.5,6,MCC7.EE4)

MFAEI4. Students will solve literal equations.

a. Solve for any variable in a multi-variable equation. (MCC6.EE.9,MCC9-12.A.REI.3)
b. Rearrange formulas to highlight a particular variable using the same reasoning as in solving equations. *For example, solve for the base in A = ½ bh.* (MCC9-12.A.CED.4)

Quantitative Reasoning with Functions

Students will create function statements and analyze relationships among pairs of variables using graphs, table, and equations.

MFAQR1. Students will understand characteristics of functions.

a. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.(MCC9-12.F.IF.1)
b. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function*. (MCC9-12.F.IF.5)
c. Graph functions using sets of ordered pairs consisting of an input and the corresponding output. (MCC8.F.1, 2)

MFAQR2. Students will compare and graph functions.

a. Calculate rates of change of functions, comparing when rates increase, decrease, or stay constant. *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* (MCC6.RP.2;MCC7.RP.1,2,3;MCC8.F.2,5; MCC9-12.F.IF.6)
b. Graph by hand simple functions expressed symbolically (use all four quadrants). (MCC9-12.F.IF.7)
c. Interpret the equation y = mx + b as defining a linear function whose graph is a straight line. (MCC8.F.3)
d. Use technology to graph non-linear functions. (MCC8.F.3, MCC9-12.F.IF.7)
e. Analyze graphs of functions for key features (intercepts, intervals of increase/decrease, maximums/minimums, symmetries, and end behavior) based on context. (MCC9-12.F.IF.4,7)

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f. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the great rate of change.* (MCC8.F.2)

MFAQR3. Students will construct and interpret functions.

a. Write a function that describes a relationship between two quantities. (MCC8.F.4, MCC9-12.F.BF.1)
b. Use variables to represent two quantities in a real-world problem that change in relationship to one another (conceptual understanding of a variable). (MCC6.EE.9)
c. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of context. (MCC9-12.F.IF.2)

Algebra:

Concepts & Connections

(HS Course 1)

MATHEMATICS
KEY COMPETENCIES &
COURSE STANDARDS

WITH
LEARNING OBJECTIVES
IN PROGRESSION ORDER

GEORGIA'S K-12 MATHEMATICS STANDARDS 2021

Governor Kemp and Superintendent Woods are committed to the best set of academic standards for Georgia's students – laying a strong foundation of the fundamentals, ensuring age- and developmentally appropriate concepts and content, providing instructional supports to set our teachers up for success, protecting and affirming local control and flexibility regarding the use of mathematical strategies and methods, and preparing students for life. These Georgia-owned and Georgia-grown standards leverage the insight, expertise, experience, and efforts of thousands of Georgians to deliver the very best educational experience for Georgia's 1.7 million students.

In August 2019, Governor Brian Kemp and State School Superintendent Richard Woods announced the review and revision of Georgia's K-12 mathematics standards. Georgians have been engaged throughout the standards review and revision process through public surveys and working groups. In addition to educator working groups, surveys, and the Academic Review Committee, Governor Kemp announced a new way for Georgians to provide input on the standards: the Citizens Review Committee, a group composed of students, parents, business and community leaders, and concerned citizens from across the state. Together, these efforts were undertaken to ensure Georgians will have buy-in and faith in the process and product.

The Citizens Review Committee provided a charge and recommendations to the working groups of educators who came together to craft the standards, ensuring the result would be usable and friendly for parents and students in addition to educators. More than 14,000 Georgians participated in the state's public survey from July through September 2019, providing additional feedback for educators to review. The process of writing the standards involved more than 200 mathematics educators -- from beginning to veteran teachers, representing rural, suburban, and metro areas of our state.

Grade-level teams of mathematics teachers engaged in deep discussions; analyzed stakeholder feedback; reviewed every single standard, concept, and skill; and provided draft recommendations. To support fellow mathematics teachers, they also developed learning progressions to show when key concepts were introduced and how they progressed across grade levels, provided examples, and defined age/developmentally appropriate expectations.

These teachers reinforced that strategies and methods for solving mathematical problems are classroom decisions -- not state decisions -- and should be made with the best interest of the individual child in mind. These recommended revisions have been shared with the Academic Review Committee, which is composed of postsecondary partners, age/development experts, and business leaders, as well as the Citizens Review Committee, for final input and feedback.

Based on the recommendation of Superintendent Woods, the State Board of Education will vote to post the draft K-12 mathematics standards for public comment. Following public comment, the standards will be recommended for adoption, followed by a year of teacher training and professional learning prior to implementation.

Algebra: Concepts & Connections

Overview

This document contains a draft of Georgia's 2021 K-12 Mathematics Standards for the High School Algebra: Concepts and Connections Course, which is the first course in the high school course sequence.

The standards are organized into big ideas, course competencies/standards, and learning objectives/expectations. The grade level key competencies represent the standard expectation of learning for students in each grade level. The competencies/standards are each followed by more detailed learning objectives that further explain the expectations for learning in the specific grade levels.

New instructional supports are included, such as clarification of language and expectations, as well as detailed examples. These have been provided for teaching professionals and stakeholders through the Evidence of Student Learning Column that accompanies each learning objective.

Course Description:

This course is designed as the first course in a three-course series. Students will apply their algebraic and geometric reasoning skills to make sense of problems involving algebra, geometry, bivariate data, and statistics. This course focuses on algebraic, quantitative, geometric, graphical, and statistical reasoning. In this course, students will continue to enhance their algebraic reasoning skills when analyzing and applying a deep understanding of linear functions, sums and products of rational and irrational numbers, systems of linear inequalities, distance, midpoint, slope, area, perimeter, nonlinear equations and functions, quadratic expressions, equations and functions, exponential expressions, equations, and functions, and statistical reasoning.

High school course content standards are listed by big ideas including Data and Statistical Reasoning, Probabilistic Reasoning, Functional and Graphical Reasoning, Patterning and Algebraic Reasoning, and Geometry Patterning and Spatial Reasoning.

Prerequisite:

This course is designed for students who have successfully completed *Kindergarten through 8th grade mathematics.*

Georgia's K-12 Mathematics Standards - 2021
Mathematics Big Ideas and Learning Progressions, High School

Mathematics Big Ideas, HS

HIGH SCHOOL
MATHEMATICAL PRACTICES (MP)
MATHEMATICAL MODELING (MM)
NUMERICAL REASONING (NR)
PATTERNING & ALGEBRAIC REASONING (PAR)
FUNCTIONAL & GRAPHICAL REASONING (FGR)
GEOMETRIC & SPATIAL REASONING (GSR)
DATA & STATISTICAL REASONING (DSR)
PROBABILISTIC REASONING (PR)

The 8 Mathematical Practices and the Mathematical Modeling Framework are essential to the implementation of the content standards presented in this course. More details related to these concepts can be found in the links below and in the first two standards presented in this course:

Mathematical Practices

Mathematical Modeling Framework

Algebra: Concepts & Connections

The eleven course standards listed below are the key content competencies students will be expected to master in this course. Additional clarity and details are provided through the classroom-level learning objectives and evidence of student learning details for each course standard found on subsequent pages of this document.

COURSE STANDARDS
A.MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.
A.MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.
A.FGR.2: Construct and interpret arithmetic sequences as functions, algebraically and graphically, to model and explain real-life phenomena. Use formal notation to represent linear functions and the key characteristics of graphs of linear functions, and informally compare linear and non-linear functions using parent graphs.
A.GSR.3: Solve problems involving distance, midpoint, slope, area, and perimeter to model and explain real-life phenomena.
A.PAR.4: Create, analyze, and solve linear inequalities in two variables and systems of linear inequalities to model real-life phenomena.
A.NR.5: Investigate rational and irrational numbers and rewrite expressions involving square roots and cube roots.
A.PAR.6: Build quadratic expressions and equations to represent and model real-life phenomena; solve quadratic equations in mathematically applicable situations.
A.FGR.7: Construct and interpret quadratic functions from data points to model and explain real-life phenomena; describe key characteristics of the graph of a quadratic function to explain a mathematically applicable situation for which the graph serves as a model.
A.PAR.8: Create and analyze exponential expressions and equations to represent and model real-life phenomena; solve exponential equations in mathematically applicable situations.
A.FGR.9: Construct and analyze the graph of an exponential function to explain a mathematically applicable situation for which the graph serves as a model; compare exponential with linear and quadratic functions.
A.DSR.10: Collect, analyze, and interpret univariate quantitative data to answer statistical investigative questions that compare groups to solve real-life problems; Represent bivariate data on a scatter plot and fit a function to the data to answer statistical questions and solve real-life problems.

Algebra: Concepts & Connections

<table>
<tr><td colspan="4">MATHEMATICAL MODELING</td></tr>
<tr><td colspan="4">A.MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.</td></tr>
<tr><th colspan="2">Expectations</th><th colspan="2">Evidence of Student Learning
(not all inclusive; see Course Overview for more details)</th></tr>
<tr><td>A.MM.1.1</td><td>Explain applicable, mathematical problems using a mathematical model.</td><td colspan="2">Fundamentals
• Students should be provided with opportunities to learn mathematics in the framework of real-life problems.
• Mathematically applicable problems are those presented in which the given framework makes sense, realistically and mathematically, and allows for students to make decisions about how to solve the problem (model with mathematics).</td></tr>
<tr><td>A.MM.1.2</td><td>Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities domains.</td><td colspan="2">Fundamentals
• Students should be able to use the content learned in this course to create a mathematical model to explain real-life phenomena.</td></tr>
<tr><td>A.MM.1.3</td><td>Use units of measure (linear, area, capacity, rates, and time) as a way to make sense of conceptual problems; identify, use, and record appropriate units of measure within the given framework, within data displays, and on graphs; convert units and rates using proportional reasoning given a conversion factor; use units within multi-step problems and formulas; interpret units of input and resulting units of output.</td><td>Strategies and Methods
• Dimensional analysis may be used when converting units and rates.</td><td>Examples
• Units of measure may include linear, area, capacity, rates, and time.</td></tr>
<tr><td>A.MM.1.4</td><td>Use various mathematical representations and structures with this information to represent and solve real-life problems.</td><td colspan="2">Strategies and Methods
• Students should be able to fluently navigate between mathematical representations that are presented numerically, algebraically, and graphically.
• For graphical representations, students should be given opportunities to analyze graphs using interactive graphing technologies.</td></tr>
<tr><td>A.MM.1.5</td><td>Define appropriate quantities for the purpose of descriptive modeling.</td><td colspan="2">Fundamentals
• Given a situation, framework, or problem, students should be able to determine, identify, and use appropriate quantities for representing the situation.</td></tr>
</table>

***FUNCTIONAL & GRAPHICAL REASONING** – function notation, modeling linear functions, linear vs. nonlinear comparisons*

A.FGR.2: Construct and interpret arithmetic sequences as functions, algebraically and graphically, to model and explain real-life phenomena. Use formal notation to represent linear functions and the key characteristics of graphs of linear functions, and informally compare linear and non-linear functions using parent graphs.

Expectations		Evidence of Student Learning **(not all inclusive; see Course Overview for more details)**	

A.FGR.2.1	Use mathematically applicable situations algebraically and graphically to build and interpret arithmetic sequences as functions whose domain is a subset of the integers.	***Fundamentals***
		• Students should be able to:
		  ○ make connections between linear functions and arithmetic sequences presented in mathematically applicable-situations.
		  ○ build and interpret arithmetic sequences as functions presented graphically and algebraically.
		  ○ convert arithmetic sequences from explicit to recursive form and vice versa.
		  ○ define sequences recursively and explicitly.
			Example
			• By graphing or calculating terms, students should be able to show how the arithmetic sequence in recursive form $a_1=7$, $a_n=a_{n-1}+2$; the arithmetic sequence in explicit form $a_n = 2(n-1) + 7$; and the function $f(x) = 2x + 5$ (when x is a natural number) all define the same sequence.

A.FGR.2.2	Construct and interpret the graph of a linear function that models real-life phenomena and represent key characteristics of the graph using formal notation.	***Strategies and Methods*** • Students should be able to use graphs created by hand and with technology, verbal descriptions, tables, and function notation when analyzing linear functions that represent real-life phenomena. • Students should be given opportunities to use interactive graphing technologies to explore and analyze key characteristics of linear functions, including domain, range, intercepts, intervals where the function is increasing or decreasing, positive or negative, maximums and minimums over a specified interval, and end behavior.	***Fundamentals*** • Students should be able to express characteristics in interval and set notation with linear functions. • Students should be able to interpret the key characteristics of the graph in a situation.

A.FGR.2.3	Relate the domain and range of a linear function to its graph and, where applicable, to the quantitative relationship it describes. Use formal interval and set notation to describe the domain and range of linear functions.	***Examples***
		• If the function h(n) gives the number of hours it takes a person to assemble n engines in a factory, then the set of positive integers would be an appropriate domain for the function.
		• Use symbolic notation to represent the domain and range of a linear function, considering the specific context.
		  ○ $(-\infty,\infty)$
		  ○ $[3, \infty)$
		  ○ D: $\{x \mid x \in \mathbb{R}\}$
		  ○ D: $\{x \mid x > 0\}$
		  ○ D: $\{x \mid x = 1,2,3,4,5,\ldots\}$
		  ○ R: $\{y \mid y = 10,20,30,\ldots\}$	

A.FGR.2.4	Use function notation to build and evaluate linear functions for inputs in their domains and interpret statements that use function	***Fundamentals*** • Student should develop a deep understanding of function notation to build, evaluate, and interpret linear functions; this understanding will be applied to other functions studied hereafter.	

	notation in terms of a mathematical framework.	• Students should be able to interpret the domain when given a function expressed numerically, algebraically, and graphically.	
A.FGR.2.5	Analyze the difference between linear functions and nonlinear functions by informally analyzing the graphs of various parent functions (linear, quadratic, exponential, absolute value, square root, and cube root parent curves).	***Fundamentals*** • Students should explore the parent function graphs to compare linear and nonlinear relationships (including a visual analysis of end behavior, increasing and decreasing, domain and range, intercepts, and general curvature). • Learning all the characteristics of these nonlinear functions is not an expectation for this learning objective. • Students should be able to identify parent functions by name (i.e., linear, quadratic, etc.). • Students should have opportunities to explore the various graphs using technology.	***Strategies and Methods*** • Students should be able to informally analyze the curvature of several parent functions to highlight the characteristics of linear functions in comparison to several nonlinear functions. • This is an introduction to functions they will explore in future units and courses. • Student should be provided opportunities to utilize graphing calculators and interactive graphing technologies to explore this concept.

***GEOMETRIC & SPATIAL REASONING** – distance, midpoint, slope, area, and perimeter*			
A.GSR.3: Solve problems involving distance, midpoint, slope, area, and perimeter to model and explain real-life phenomena.			
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**	
A.GSR.3.1	Solve real-life problems involving slope, parallel lines, perpendicular lines, area, and perimeter.	***Fundamentals*** • Students should apply their understanding of linear relationships to solve real-life, application problems related to slope, parallel lines, perpendicular lines, area, and perimeter. • Students should be able to calculate the area and perimeter of special parallelograms and triangles with simple, unknown side lengths.	
A. GSR.3.2	Apply the distance formula, midpoint formula, and slope of line segments to solve real-world problems.	***Fundamentals*** • Students should be able to apply their understanding of slope and use the distance and midpoint formulas to solve real-world problems. • In a real-life application, using a figure in the coordinate plane, students should be able to find a location using distance or midpoint.	***Example*** • Find the distance of a line segment plotted on the coordinate plane.

PATTERNING & ALGEBRAIC REASONING – linear inequalities and systems of linear inequalities			
A.PAR.4: Create, analyze, and solve linear inequalities in two variables and systems of linear inequalities to model real-life phenomena.			
Expectations		**Evidence of Student Learning** (not all inclusive; see Course Overview for more details)	
A.PAR.4.1	Create and solve linear inequalities in two variables to represent relationships between quantities including mathematically applicable situations; graph inequalities on coordinate axes with labels and scales.	***Fundamentals*** • Students should be given the opportunity to explore the difference between solid lines and dashed lines through exploration on an interactive graph. • Students should have had opportunities to create and solve linear equations and inequalities throughout middle school mathematics. • Students should recognize that the graph of a linear inequality in two variables is a half-plane.	***Strategies and Methods*** • When necessary, students should be able to rewrite the inequality in various forms, such as slope-intercept form, for graphing. • Students should be given opportunities to solve linear inequalities graphically and algebraically. These linear inequalities should represent realistic, real-life phenomena.
A.PAR.4.2	Represent constraints of linear inequalities and interpret data points as possible or not possible.	***Terminology*** • Possible data points are solutions to the inequality or inequalities; data points that are not possible are non-solutions to the inequality or inequalities.	
A.PAR.4.3	Solve systems of linear inequalities by graphing, including systems representing a mathematically applicable situation.	***Fundamentals*** • Ensure constraints are represented. • Students in Grade 8 mathematics modeled with and solved systems of linear equations to solve real-life problems.	***Strategies and Methods*** • Students should be provided opportunities to use technology tools to solve systems of linear inequalities graphically.

NUMERICAL REASONING - rational and irrational numbers, square roots and cube roots			
A.NR.5: Investigate rational and irrational numbers and rewrite expressions involving square roots and cube roots.			
Expectations		**Evidence of Student Learning** (not all inclusive; see Course Overview for more details)	
A.NR.5.1	Rewrite algebraic and numeric expressions involving radicals.	***Relevance and Application*** • Students should be able to use the operations of addition, subtraction, and multiplication, with radicals within expressions limited to square roots and cube roots.	
A.NR.5.2	Using numerical reasoning, show and explain that the sum or product of rational numbers is rational, the sum of a rational number and an irrational number is irrational, and the product of a nonzero rational number and an irrational number is irrational.	**Fundamentals** • The tasks selected should aid students with their development of a conceptual understanding of the sums and products of rational and irrational numbers through exploration and investigation. • Students should be able to judge the reasonableness of an answer based on their understanding of rational and irrational numbers.	**Examples** • Students should know that adding two irrational numbers, such as $3\sqrt{5}$ and $\sqrt{7}$, may result in an irrational number. • The side length of a square is $\sqrt{8}$. Is the perimeter a rational or irrational number?

<table>
<tr><td colspan="4">PATTERNING & ALGEBRAIC REASONING – quadratic expressions & equations</td></tr>
<tr><td colspan="4">A.PAR.6: Build quadratic expressions and equations to represent and model real-life phenomena; solve quadratic equations in mathematically applicable situations.</td></tr>
<tr><td colspan="2">Expectations</td><td colspan="2">Evidence of Student Learning
(not all inclusive; see Course Overview for more details)</td></tr>
<tr><td>A.PAR.6.1</td><td>Interpret quadratic expressions and parts of a quadratic expression that represent a quantity in terms of its context.</td><td colspan="2">Fundamentals
• Students should be able to interpret parts of an expression, such as terms, factors, leading coefficient, coefficients, constant and degree in context.
• Given mathematically applicable situations which utilize formulas or expressions with multiple terms and/or factors, students should be able to interpret the meaning of given individual terms or factors.</td></tr>
<tr><td>A.PAR.6.2</td><td>Fluently choose and produce an equivalent form of a quadratic expression to reveal and explain properties of the quantity represented by the expression.</td><td>Fundamentals
• Students should be able to multiply variable expressions involving the product of a monomial and a binomial and the product of two binomials to produce a quadratic expression.
• Polynomial operations are included with this objective. Polynomial sums, differences, and products should not exceed a maximum degree of 2.</td><td>Strategies and Methods
• Students should be able to move fluently (flexibly, accurately, efficiently) between different forms of a quadratic expression (standard, vertex, and factored forms).
• Students should be able to use the structure of a quadratic expression to rewrite it in different equivalent forms.</td></tr>
<tr><td>A.PAR.6.3</td><td>Create and solve quadratic equations in one variable and explain the solution in the framework of applicable phenomena.</td><td colspan="2">Fundamentals
• Students should be able to multiply variable expressions involving the product of a monomial and a binomial and the product of two binomials to solve a quadratic equation.

Strategies and Methods
• Students should be able to solve quadratic equations fluently (flexibly, accurately, efficiently) by inspection, taking square roots, factoring, completing the square, and applying the quadratic formula, as appropriate to the initial form of the equation.
• Students should be able to fluently transform a quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions.
• Students should be able to analyze and explain what the zeros describe in context.

Relevance and Application
• Limit to real number solutions.</td></tr>
<tr><td>A.PAR.6.4</td><td>Represent constraints by quadratic equations and interpret data points as possible or not possible in a modeling framework.</td><td colspan="2">Terminology
• Possible data points are solutions to the equation(s); data points that are not possible are non-solutions to the equation(s).</td></tr>
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<table>
<tr><td colspan="2">FUNCTIONAL & GRAPHICAL REASONING – quadratic functions</td></tr>
<tr><td colspan="2">A.FGR.7: Construct and interpret quadratic functions from data points to model and explain real-life phenomena; describe key characteristics of the graph of a quadratic function to explain a mathematically applicable situation for which the graph serves as a model.</td></tr>
<tr><td colspan="2">Expectations</td><td>Evidence of Student Learning
(not all inclusive; see Course Overview for more details)</td></tr>
<tr><td>A.FGR.7.1</td><td>Use function notation to build and evaluate quadratic functions for inputs in their domains and interpret statements that use function notation in terms of a given framework.</td><td>Fundamentals
• Students should apply their understanding of function notation from their work with linear functions to build, evaluate, and interpret quadratic functions using function notation.
• Students should be able to interpret the domain given a function expressed numerically, algebraically, and graphically.</td></tr>
<tr><td>A.FGR.7.2</td><td>Identify the effect on the graph generated by a quadratic function when replacing f(x) with f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs.</td><td>Strategies and Methods
• Students should be given opportunities to experiment with cases and illustrate an explanation of the effects on the graph using technology.</td></tr>
<tr><td>A.FGR.7.3</td><td>Graph and analyze the key characteristics of quadratic functions.</td><td>Strategies and Methods
• Students should be able to use verbal descriptions, tables, and graphs created using interactive technology tools.

Fundamentals
• Students should be able to sketch a graph showing key features including domain, range, and intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; asymptotes; end behavior.
• Key characteristics of the quadratic functions should be expressed in interval and set-builder notation using inequalities.</td></tr>
<tr><td>A.FGR.7.4</td><td>Relate the domain and range of a quadratic function to its graph and, where applicable, to the quantitative relationship it describes.</td><td>Examples
• If the function h(t) gives the path of a projectile over time, t, then the set of non-negative real numbers would be an appropriate domain for the function because time does not include negative values.
• A bird is building a nest in a tree 36 feet above the ground. The bird drops a stick from the nest. The function $f(x) = -16x^2 + 36$ describes the height of the stick in feet after x seconds. Graph this function. Identify the domain and range of this function. (A student should be able to determine that the appropriate values for the domain and range of this graph are $0 \le x \le 1.5$ and $0 \le y \le 36$, respectively.)</td></tr>
<tr><td>A.FGR.7.5</td><td>Rewrite a quadratic function representing a mathematically applicable situation to reveal the maximum or minimum value of the function it defines. Explain what the value describes in context.</td><td>Fundamentals
• Students should be able to interpret the maximum and minimum value of a quadratic function expressed in a variety of ways.

Strategies and Methods
• Students should be able to use interactive graphing technologies to make sense of the maximum and minimum values in context.

Example
• Consider the path of a football thrown through the air. When does the football reach its maximum height? How high does the football reach?</td></tr>
</table>

A.FGR.7.6	Create quadratic functions in two variables to represent relationships between quantities; graph quadratic functions on the coordinate axes with labels and scales.	***Strategies and Methods*** • Students should be able to use interactive graphing technologies to make sense of the visual, graphical model for a quadratic function representing a mathematically applicable situation.
A.FGR.7.7	Estimate, calculate, and interpret the average rate of change of a quadratic function and make comparisons to the average rate of change of linear functions.	***Fundamentals*** • Students should be given opportunities to estimate the rate of change from a graph. • Students should be able to show that linear functions grow by equal differences over equal intervals and recognize situations in which one quantity changes at a constant rate per unit interval relative to another. Students should be able to compare this behavior to that of the average rate of change of quadratic functions. This can be shown by algebraic proof, with a table showing differences, or by calculating average rates of change over equal intervals. ***Strategies and Methods*** • Functions can be presented symbolically, as a graph, or as a table.

A.FGR.7.8	Write a function defined by a quadratic expression in different but equivalent forms to reveal and explain different properties of the function.	***Strategies and Methods***
		• Students should be able to move fluently (flexibly, accurately, efficiently) between the factored form, vertex form, and standard form of a quadratic function.
		
		Fundamentals
		• Students should be able to examine a quadratic function by analyzing the zeros, extreme values, and symmetry of the graph and interpret these properties in context.
		
		Strategies and Methods
		• Students should be given opportunities to use a variety of strategies and methods to make sense of the properties of quadratic functions:
		  ○ Factoring
		  ○ Completing the square
		  ○ Quadratic formula
		  ○ Graphing
		  ○ Taking square roots
		
		Example
		• Students should be able to compare quadratic functions in standard, vertex, and intercept forms.

A.FGR.7.9	Compare characteristics of two functions each represented in a different way.	***Fundamentals***
		• Functions can be presented numerically in tables, algebraically, graphically, and by verbal descriptions.
		• Students should be able to:
		  ○ compare a quadratic function to a linear function, or another quadratic function.
		  ○ compare key characteristics of quadratic functions with the key characteristics of linear functions.
		  ○ observe using graphs and tables that a quantity increasing quadratically will eventually exceed a portion of a quantity increasing linearly.
		
		Examples
		• Given a graph of one quadratic function and an algebraic equation for another, students should be able to determine which has the larger maximum.
		• Given a graph of one function and an algebraic equation for another, students should be able to determine which has the larger y-intercept.

PATTERNING & ALGEBRAIC REASONING – *exponential expressions and equations*		
A.PAR.8: Create and analyze exponential expressions and equations to represent and model real-life phenomena; solve exponential equations in mathematically applicable situations.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
A.PAR.8.1	Interpret exponential expressions and parts of an exponential expression that represent a quantity in terms of its framework.	***Fundamentals*** • Students should be able to interpret parts of an expression, such as terms, factors, leading coefficient, coefficients, constant and degree in context. • Given mathematically applicable situations which utilize formulas or expressions with multiple terms and/or factors, students should be able to interpret the meaning in context of individual terms or factors.
A.PAR.8.2	Create exponential equations in one variable and use them to solve problems, including mathematically applicable situations.	***Relevance and Application*** • Exponential equations are limited to those containing like bases, or exponential equations that could easily be transferred to like bases with linear operations.
A.PAR.8.3	Create exponential equations in two variables to represent relationships between quantities, including in mathematically applicable situations; graph equations on coordinate axes with labels and scales.	***Example*** • Exponential growth and decay situations are an expectation for this learning objective.
A.PAR.8.4	Represent constraints by exponential equations and interpret data points as possible or not possible in a modeling environment.	***Terminology*** • Possible data points are solutions to the equation(s); data points that are not possible are non-solutions to the equation(s).

FUNCTIONAL & GRAPHICAL REASONING – *exponential functions*		
A.FGR.9: Construct and analyze the graph of an exponential function to explain a mathematically applicable situation for which the graph serves as a model; compare exponential with linear and quadratic functions.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
A.FGR.9.1	Use function notation to build and evaluate exponential functions for inputs in their domains and interpret statements that use function notation in terms of a context.	***Fundamentals*** • Students should apply their understanding of function notation from their work with linear and quadratic functions to build, evaluate, and interpret exponential functions using function notation. • Students should be able to interpret the domain given a function expressed numerically, algebraically, and graphically.
A.FGR.9.2	Graph and analyze the key characteristics of simple exponential functions based on mathematically applicable situations.	***Examples*** • If the function, *h(n),* gives the number of person-hours it takes to assemble *n* engines in a factory, then the positive integers would be an appropriate domain for the function. • The function can be presented symbolically, as a graph, or as a table. • Students should be able to estimate the rate of change from a graph.

		• Students should be able to sketch a graph of an exponential function showing key features including domain, range, intercepts, average rate of change, intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; asymptotes; end behavior. • Students should be given opportunities to show that linear functions grow by a constant rate and that exponential functions grow by equal factors over equal intervals. This can be shown by algebraic proof, with a table showing differences, or by calculating average rates of change over equal intervals. • Students should be able to precisely use verbal descriptions, tables, and graphs created by hand and using technology. • Students should be able to create graphs by hand and using graphing technology (i.e., graphing calculator or online interactive graphing technology) • Students should be able to accurately express characteristics in interval notation and set-builder notation using inequalities.	
A.FGR.9.3	Identify the effect on the graph generated by an exponential function when replacing f(x) with f(x) + k, and k f(x), for specific values of k (both positive and negative); find the value of k given the graphs.	***Strategies and Methods*** • Students should be given opportunities to experiment with cases and illustrate an explanation of the effects on the graph using interactive technology.	
A.FGR.9.4	Use mathematically applicable situations algebraically and graphically to build and interpret geometric sequences as functions whose domain is a subset of the integers.	***Fundamentals*** • Sequences can be defined recursively and explicitly. • Connections should be made between exponential functions and geometric sequences. • The focus of this learning objective is on building and interpreting geometric sequences. • Students should be able to covert geometric sequences from explicit form to recursive and vice versa. • Students should have ample opportunities to compare geometric sequences with arithmetic sequences presented in a variety of ways.	***Example*** • By graphing or calculating terms, students should be able to show how the geometric sequence in recursive form $a_1=8$, $a_n=2a_{n-1}$; the geometric sequence in explicit form $s_n = 8(2)^{n-1}$; and the function $f(x) = 4(2)^x$ (when x is a natural number) all define the same sequence.
A.FGR.9.5	Compare characteristics of two functions each represented in a different way.	***Fundamentals*** • Students should be able to present functions algebraically, graphically, and numerically in tables, or by verbal descriptions. • Students should be able to compare an exponential function to a linear function, a quadratic function, or to another exponential function. • Students should be able to compare key characteristics of exponential functions with the key characteristics of linear and quadratic functions. • Students should be able to observe using graphs and tables that a quantity increasing quadratically will eventually exceed a portion of a quantity increasing linearly. • Students should be able to observe using graphs and tables that a quantity increasing exponentially will eventually exceed a portion of a quantity increasing linearly or quadratically.	***Example*** • Given a graph of one function and an algebraic expression for another, determine which has the larger y-intercept.

DATA & STATISTICAL REASONING – *univariate data and single quantitative variables; bivariate data*			
A.DSR.10: Collect, analyze, and interpret univariate quantitative data to answer statistical investigative questions that compare groups to solve real-life problems; Represent bivariate data on a scatter plot and fit a function to the data to answer statistical questions and solve real-life problems.			
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**	
A.DSR.10.1	Use statistics appropriate to the shape of the data distribution to compare and represent center (median and mean) and variability (interquartile range, standard deviation) of two or more distributions by hand and using technology.	***Terminology*** • Measures of center include the median and mean. • Measures of spread include the range, interquartile range and standard deviation. • Univariate data involves describing a single variable, such as the age of a student or the height of a student. • Bivariate data involves relationships between two variables, such as comparing the age of a student and their height.	***Fundamentals*** • Students should use the meaning of mean absolute deviation (MAD) learned in sixth grade to interpret the meaning of standard deviation. • Students were first introduced to the concept of MAD as a tool for comparing variability of multiple data sets in sixth grade mathematics. • Students should initially have opportunities to explore standard deviation, by hand, with small data sets, to gain conceptual understanding. • Students should advance to using technology to determine standard deviation to solve problems and answers statistical investigative questions.
A.DSR.10.2	Interpret differences in shape, center, and variability of the distributions based on the investigation, accounting for possible effects of extreme data points (outliers).	***Strategies and Methods*** • Use the 1.5 IQR rule to determine the outliers and analyze their effects on the data set.	***Example*** • Using the 1.5 IQR rule on data set {5,7,8,10,11,12,30}, 30 is determined to be an outlier since it is greater than 19.5, which is the 1.5*IQR +12 (the Q3).
A.DSR.10.3	Represent data on two quantitative variables on a scatter plot and describe how the variables are related.	***Fundamentals*** • Students should be able to describe the direction, strength, and form (linear, non-linear) of the association between two quantitative variables.	
A.DSR.10.4	Interpret the slope (predicted rate of change) and the intercept (constant term) of a linear model based on the-investigation of the data.	***Strategies and Methods*** • Students should be given the opportunity to utilize interactive graphing technologies to model linear data and make sense of the slope (predicted rate of change) visually.	
A.DSR.10.5	Calculate the line of best fit and interpret the correlation coefficient, *r,* of a linear fit using technology. Use *r* to describe the strength of the goodness of fit of the regression. Use the linear function to make predictions and	***Strategies and Methods*** • Students should be given the opportunity to utilize interactive graphing technologies to interpret the correlation coefficient, *r*.	***Fundamentals*** • Students should be able to use the line of best fit and the correlation coefficient, *r*, to make predictions and describe the reasonableness of the prediction in the investigation of a practical, real-life situation.

	assess how reasonable the prediction is in context.		
A.DSR.10.6	Decide which type of function is most appropriate by observing graphed data.	***Fundamentals*** • Students should be able to emphasize linear, quadratic, and exponential models.	
A.DSR.10.7	Distinguish between correlation and causation.	***Application and Relevance*** • It is important for students to discover and understand that strong association does not indicate causation.	

ESSENTIAL INSTRUCTIONAL GUIDANCE

MATHEMATICAL PRACTICES

The Mathematical Practices describe the reasoning behaviors students should develop as they build an understanding of mathematics – the "habits of mind" that help students become mathematical thinkers. There are eight standards, which apply to all grade levels and conceptual categories.

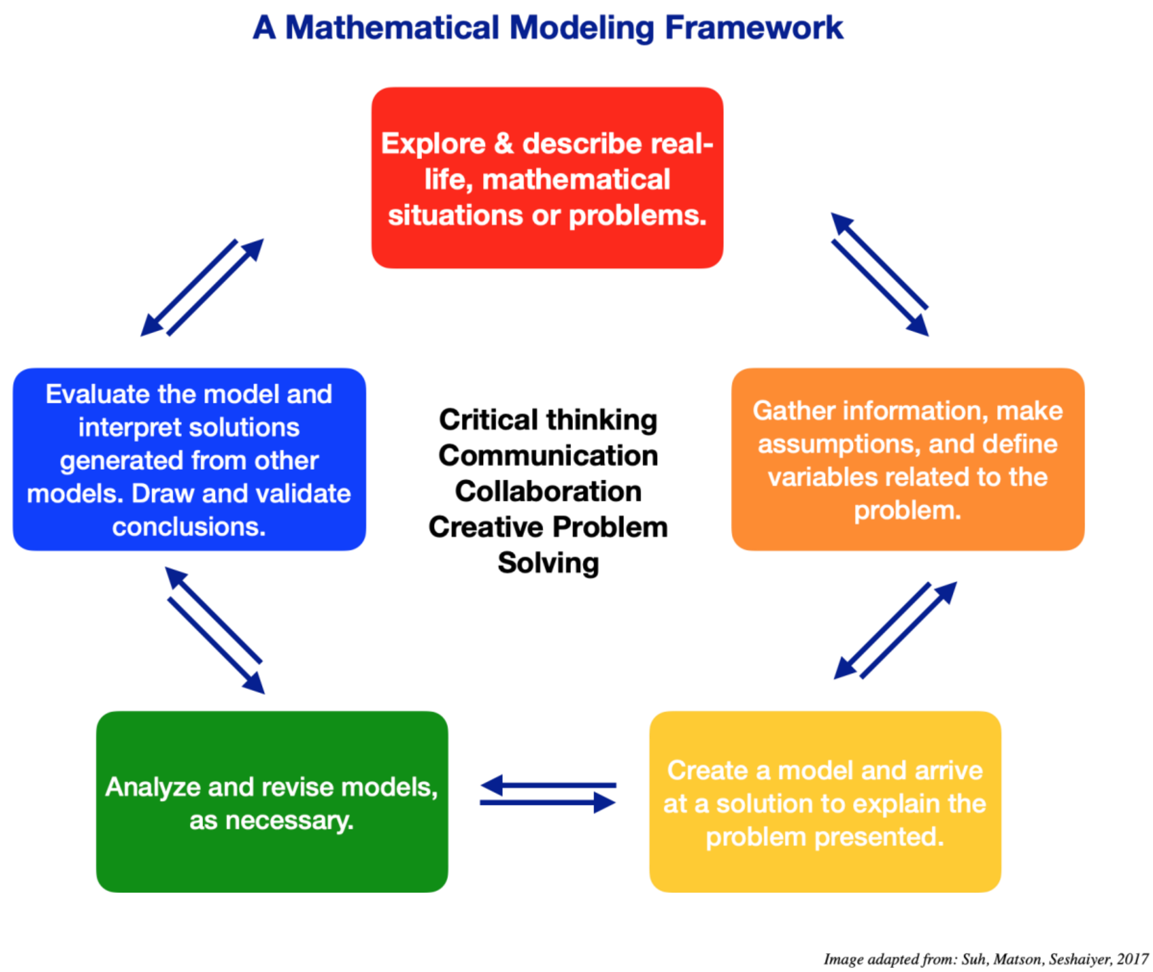
These mathematical practices describe how students should engage with the mathematics content for their grade level. Developing these habits of mind builds students' capacity to become mathematical thinkers. These practices can be applied individually or together in mathematics lessons, and no particular order is required. In well-designed lessons, there are often two or more Standards for Mathematical Practice present.

MATHEMATICAL PRACTICES	
A.MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.	
Code	**Expectation**
A.MP.1	Make sense of problems and persevere in solving them.
A.MP.2	Reason abstractly and quantitatively.
A.MP.3	Construct viable arguments and critique the reasoning of others.
A.MP.4	Model with mathematics.
A.MP.5	Use appropriate tools strategically.
A.MP.6	Attend to precision.
A.MP.7	Look for and make use of structure.
A.MP.8	Look for and express regularity in repeated reasoning.

MATHEMATICAL MODELING

Teaching students to model with mathematics is engaging, builds confidence and competence, and gives students the opportunity to collaborate and make sense of the world around them, the main reason for doing mathematics. For these reasons, mathematical modeling should be incorporated at every level of a student's education. This is important not only to develop a deep understanding of mathematics itself, but more importantly to give students the tools they need to make sense of the world around them. Students who engage in mathematical modeling will not only be prepared for their chosen career but will also learn to make informed daily life decisions based on data and the models they create.

The diagram below is a mathematical modeling framework depicting a cycle of how students can engage in mathematical modeling when solving a real-life problem or task.



FRAMEWORK FOR STATISTICAL REASONING

Statistical reasoning is important for learners to engage as citizens and professionals in a world that continues to change and evolve. Humans are naturally curious beings and statistics is a language that can be used to better answer questions about personal choices and/or make sense of naturally occurring phenomena. Statistics is a way to ask questions, explore, and make sense of the world around us.

The Framework for Statistical Reasoning should be used in all grade levels and courses to guide learners through the sense-making process, ultimately leading to the goal of statistical literacy in all grade levels and courses. Reasoning with statistics provides a context that necessitates the learning and application of a variety of mathematical concepts.

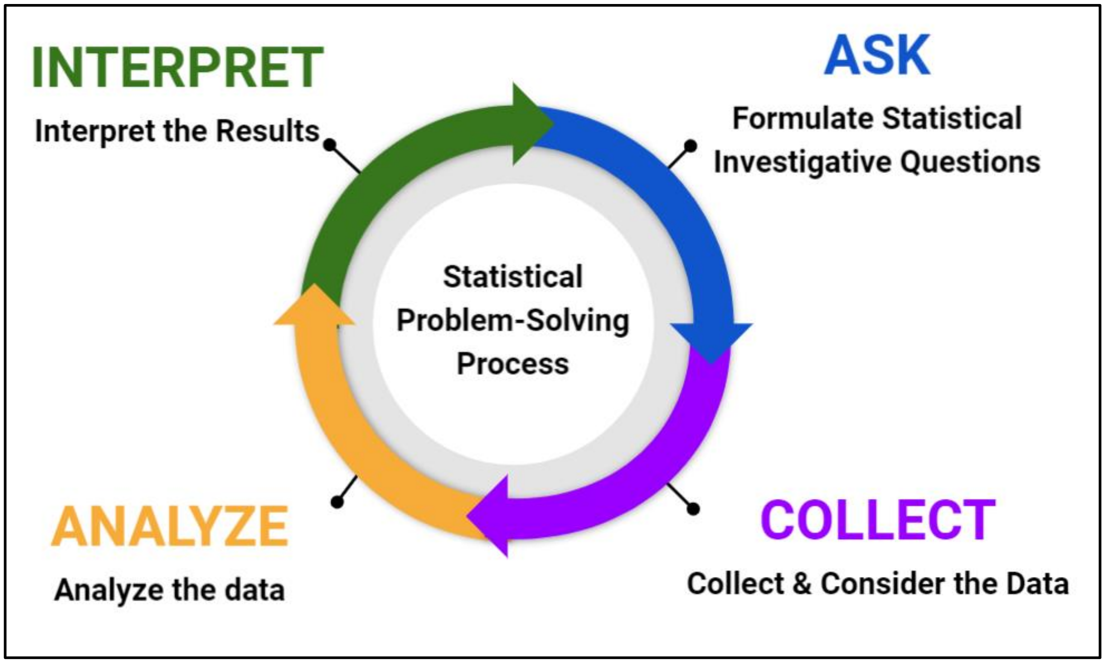


Figure 1: Georgia Framework for Statistical Reasoning

The following four-step statistical problem-solving process can be used throughout each grade level and course to help learners develop a solid foundation in statistical reasoning and literacy:

I. **Formulate Statistical Investigative Questions**
   Ask questions that anticipate variability.

II. **Collect & Consider the Data**
   Ensure that data collection designs acknowledge variability.

III. **Analyze the Data**
   Make sense of data and communicate what the data mean using pictures (graphs) and words. Give an accounting of variability, as appropriate.

IV. **Interpret the Results**
   Answer statistical investigative questions based on the collected data.