GEORGIA'S K-12
MATHEMATICS STANDARDS
2021

Geometry:

Concepts & Connections

(HS Course 2)

MATHEMATICS
KEY COMPETENCIES &
COURSE STANDARDS

WITH
LEARNING OBJECTIVES
IN PROGRESSION ORDER

GEORGIA'S K-12 MATHEMATICS STANDARDS 2021

Governor Kemp and Superintendent Woods are committed to the best set of academic standards for Georgia's students – laying a strong foundation of the fundamentals, ensuring age- and developmentally appropriate concepts and content, providing instructional supports to set our teachers up for success, protecting and affirming local control and flexibility regarding the use of mathematical strategies and methods, and preparing students for life. These Georgia-owned and Georgia-grown standards leverage the insight, expertise, experience, and efforts of thousands of Georgians to deliver the very best educational experience for Georgia's 1.7 million students.

In August 2019, Governor Brian Kemp and State School Superintendent Richard Woods announced the review and revision of Georgia's K-12 mathematics standards. Georgians have been engaged throughout the standards review and revision process through public surveys and working groups. In addition to educator working groups, surveys, and the Academic Review Committee, Governor Kemp announced a new way for Georgians to provide input on the standards: the Citizens Review Committee, a group composed of students, parents, business and community leaders, and concerned citizens from across the state. Together, these efforts were undertaken to ensure Georgians will have buy-in and faith in the process and product.

The Citizens Review Committee provided a charge and recommendations to the working groups of educators who came together to craft the standards, ensuring the result would be usable and friendly for parents and students in addition to educators. More than 14,000 Georgians participated in the state's public survey from July through September 2019, providing additional feedback for educators to review. The process of writing the standards involved more than 200 mathematics educators -- from beginning to veteran teachers, representing rural, suburban, and metro areas of our state.

Grade-level teams of mathematics teachers engaged in deep discussions; analyzed stakeholder feedback; reviewed every single standard, concept, and skill; and provided draft recommendations. To support fellow mathematics teachers, they also developed learning progressions to show when key concepts were introduced and how they progressed across grade levels, provided examples, and defined age/developmentally appropriate expectations.

These teachers reinforced that strategies and methods for solving mathematical problems are classroom decisions -- not state decisions -- and should be made with the best interest of the individual child in mind. These recommended revisions have been shared with the Academic Review Committee, which is composed of postsecondary partners, age/development experts, and business leaders, as well as the Citizens Review Committee, for final input and feedback.

Based on the recommendation of Superintendent Woods, the State Board of Education will vote to post the draft K-12 mathematics standards for public comment. Following public comment, the standards will be recommended for adoption, followed by a year of teacher training and professional learning prior to implementation.

Geometry: Concepts & Connections

Overview

This document contains a draft of Georgia's 2021 K-12 Mathematics Standards for the High School Geometry: Concepts and Connections Course, which is the second course in the high school course sequence.

The standards are organized into big ideas, course competencies/standards, and learning objectives/expectations. The grade level key competencies represent the standard expectation of learning for students in each grade level. The competencies/standards are each followed by more detailed learning objectives that further explain the expectations for learning in the specific grade levels.

New instructional supports are included, such as clarification of language and expectations, as well as detailed examples. These have been provided for teaching professionals and stakeholders through the Evidence of Student Learning Column that accompanies each learning objective.

Course Description:

This course is designed as the second course in a three-course series. This course enhances students' geometric, algebraic, graphical, and probabilistic reasoning skills. Students will apply their algebraic and geometric reasoning skills to make sense of problems involving geometry, trigonometry, algebra, probability, and statistics. Students will continue to enhance their analytical geometry and reasoning skills when analyzing and applying a deep understanding of polynomial expressions, proofs, constructions, rigid motions and transformations, similarity, congruence, circles, right triangle trigonometry, geometric measurement, and conditional probability.

High school course content standards are listed by big ideas including Data and Statistical Reasoning, Probabilistic Reasoning, Functional and Graphical Reasoning, Patterning and Algebraic Reasoning, and Geometry Patterning and Spatial Reasoning.

Course Prerequisite:

This course is designed for students who have successfully completed *Algebra: Concepts & Connections*.

Georgia's K-12 Mathematics Standards - 2021
Mathematics Big Ideas and Learning Progressions, High School

Mathematics Big Ideas, HS

HIGH SCHOOL
MATHEMATICAL PRACTICES (MP)
MATHEMATICAL MODELING (MM)
NUMERICAL REASONING (NR)
PATTERNING & ALGEBRAIC REASONING (PAR)
FUNCTIONAL & GRAPHICAL REASONING (FGR)
GEOMETRIC & SPATIAL REASONING (GSR)
DATA & STATISTICAL REASONING (DSR)
PROBABILISTIC REASONING (PR)

The 8 Mathematical Practices and the Mathematical Modeling Framework are essential to the implementation of the content standards presented in this course. More details related to these concepts can be found in the links below and in the first two standards presented in this course:

Mathematical Practices

Mathematical Modeling Framework

Geometry: Concepts & Connections

The twelve course standards listed below are the key content competencies students will be expected to master in this course. Additional clarity and details are provided through the classroom-level learning objectives and evidence of student learning details for each course standard found on subsequent pages of this document.

COURSE STANDARDS
G.MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.
G.MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.
G.PAR.2: Interpret the structure of polynomial expressions and perform operations with polynomials within a geometric framework.
G.GSR.3: Experiment with transformations in the plane to develop precise definitions for translations, rotations, and reflections and use these to describe symmetries and congruence to model and explain real-life phenomena.
G.GSR.4: Establish facts between angle relations and generate valid arguments to defend facts established. Prove theorems and solve geometric problems involving lines and angles to model and explain real-life phenomena.
G.GSR.5: Describe dilations in terms of center and scale factor and use these terms to describe properties of dilations; use the precise definition of a dilation to describe similarity and establish the criterion for triangles to be similar; use these terms, definitions, and criterion to prove similarity, model, and explain real-life phenomena.
G.GSR.6: Examine side ratios of similar triangles; use the relationship between right triangles to develop an understanding of sine and cosine to solve geometric problems and to model and explain real-life phenomena.
G.GSR.7: Explore the concept of a radian measure and special right triangles.
G.GSR.8: Examine and apply theorems involving circles; describe and derive arc length and area of a sector; and model and explain real-life situations involving circles.
G.GSR.9: Develop informal arguments for geometric formulas using dissection arguments, limit arguments, and Cavalieri's principle; solve realistic problems involving volume; explore and visualize relationships between two-dimensional and three-dimensional objects to model and explain real-life phenomena.
G.PR.10: Solve problems involving the probability of compound events to make informed decisions; interpret expected value and measures of variability to analyze probability distributions.
G.DSR.11: Examine real-life situations presented in a two-way frequency table to calculate probabilities, to model categorical data, and to explain real-life phenomena.

Geometry: Concepts & Connections

MATHEMATICAL MODELING		
G.MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.		
Expectations		**Evidence of Student Learning** (not all inclusive; see Course Overview for more details)
G.MM.1.1	Explain mathematically applicable problems using a mathematical model.	***Fundamentals*** • Students should be provided with opportunities to learn mathematics through the exploration of real-life problems. • Mathematically applicable problems are those presented in context where the context makes sense, realistically and mathematically, and allows for students to make decisions about how to solve the problem (model with mathematics).
G.MM.1.2	Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts.	***Fundamentals*** • Students should be able to use the content learned in this course to create a mathematical model to explain real-life phenomena.
G.MM.1.3	Using abstract and quantitative reasoning, make decisions about information and data from a mathematically applicable situation.	***Fundamentals*** • Students should be able to connect learning of geometric shapes and their properties to describe objects. • Students should be able to apply geometric methods and data to make decisions about structures and solve real-world problems.
G.MM.1.4	Use various mathematical representations and structures with this information to represent and solve real-life problems.	***Fundamentals*** • Students should be able to construct a model by selecting and creating algebraic and geometric representations that describe relationships between variables in context.

***PATTERNING & ALGEBRAIC REASONING** – polynomial expressions*				
G.PAR.2: Interpret the structure of and perform operations with polynomials within a geometric framework.				
	Expectations	**Evidence of Student Learning** (not all inclusive; see Course Overview for more details)		
G.PAR.2.1	Interpret polynomial expressions of varying degrees that represent a quantity in terms of its given geometric framework.	***Fundamentals*** • In Grade 8, students begin to interpret algebraic expressions and parts of an expression in context. • Students should be able to interpret parts of an expression, such as	***Relevance and Application*** • Students should have opportunities to use polynomial expressions within the context of geometric shapes.	***Example*** • Jax wants to buy a frame for an 8in x 10in photo. The frame will be the same thickness on all four sides. Write an expression to represent the perimeter and area of the frame.

		terms, factors, leading coefficient, coefficients, constant and degree in context. • Given mathematically applicable situations, which utilize formulas or expressions with multiple terms and/or factors, students should be able to interpret the meaning of individual terms or factors within the given framework.		Possible solution: A=(2x+8)(2x+10), where *x* represents the width of the frame. • Students should be able to discuss the meaning of the variable in context, the degrees of the expressions and the sums/products.
G.PAR.2.2	Perform operations with polynomials and prove that polynomials form a system analogous to the integers in that they are closed under these operations.	***Fundamentals*** • Students should understand that polynomials, like integers, are "closed" when it comes to addition, subtraction, and multiplication. • Through investigation and exploration, students should be given opportunities to discover that the sum and/or difference of two or more polynomials is a polynomial and the product of two polynomials is a polynomial. • Students should have opportunities to perform operations with binomials, trinomials, and other polynomials.		***Terminology*** • A polynomial is any expression that is a combination of one or more monomials connected via addition or subtraction.
G.PAR.2.3	Using algebraic reasoning, add, subtract, and multiply single variable polynomials.	***Fundamentals*** • Students should be able to use algebraic reasoning to show and explain how integers and polynomials are similar in that they both are closed under addition, subtraction, and multiplication. • Students should have opportunities to perform operations with first, second, third, fourth, and fifth degree polynomials.		

GEOMETRIC & SPATIAL REASONING – *congruence*		
G.GSR.3: Experiment with transformations in the plane to develop precise definitions for translations, rotations, and reflections and use these to describe symmetries and congruence to model and explain real-life phenomena.		
	Expectations	**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
G.GSR.3.1	Use geometric reasoning and symmetries of regular polygons to develop definitions of rotations, reflections, and translations.	***Fundamentals*** • Students should be able to define and identify figures as preimages and images. • Students should be provided with multiple opportunities to identify lines of symmetry and angles of rotation to map a figure onto itself.

		• Students should be provided with multiple opportunities to identify angles of rotation, lines of reflection, and directions of translations to map a preimage onto its image. • Students should be provided opportunities to experiment with transformations represented on and off the coordinate plane.
G.GSR.3.2	Verify experimentally the congruence properties of rotations, reflections, and translations: lines are taken to lines and line segments to line segments of the same length; angles are taken to angles of the same measure; parallel lines are taken to parallel lines.	***Fundamentals*** • Students should be able to determine that translations, reflections, and rotations produce images of the same size and shape as the preimage. • Students should be able to determine congruency by identifying the rigid transformation(s) that produced the image of a figure. • Opportunities should be provided for students to write statements of congruency. ***Terminology*** • A transformation that preserves size and shape is called a rigid transformation. ***Strategies and Methods*** • Students should have ample opportunities to use geometric tools and/or technology to explore figures created from translations, reflections, and rotations.
G.GSR.3.3	Use geometric descriptions of rigid motions to draw the transformed figures and to predict the effect on a given figure. Describe a sequence of transformations from one figure to another and use transformation properties to determine congruence.	***Fundamentals*** • Students should be given multiple opportunities to identify resulting coordinates from translations, reflections, and rotations, and recognize the relationship between the coordinates and the transformation. • Given two figures, students should be able to use the definition of congruence in terms of rigid motions to verify congruence if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. • Students should be able to use function notation to represent transformations in the coordinate plane. ***Strategies and Methods*** • Reflections should be limited to those over the x and y axes, horizontal and vertical lines, and the line y = x. • Rotations should be limited to those centered about the origin and in increments of 90 degrees, clockwise and counterclockwise. ***Example*** • The function notation (x, y) --> (x-4, y+5) translates the point (x, y) four units to the left and five units up.

G.GSR.3.4	Explain how the criteria for triangle congruence follow from the definition of congruence in terms of rigid motions. Use congruency criteria for triangles to solve problems and to prove relationships in geometric figures.	***Fundamentals*** • Students should be able to apply properties of congruence to solve problems with missing values involving corresponding parts. • Students should be able to use the definition of congruence to prove relationships in geometric figures.	***Strategies and Methods*** • Students should be provided opportunities to use ASA, SAS, SSS, AAS, and HL congruence postulates/theorems to prove triangles are congruent. • Students should have opportunities to prove triangle congruence using appropriate methods: logic statements, two-column proofs, paragraph proofs, and flow proofs.	***Terminology*** • Logic statements include conditional, converse, inverse, contrapositive, and conditional statements.

<table>
<tr><td colspan="4">GEOMETRIC & SPATIAL REASONING – geometric foundations, constructions, and proof</td></tr>
<tr><td colspan="4">G.GSR.4: Establish facts between angle relations and generate valid arguments to defend facts established. Prove theorems and solve geometric problems involving lines and angles to model and explain real-life phenomena.</td></tr>
<tr><td></td><td>Expectations</td><td colspan="2">Evidence of Student Learning
(not all inclusive; see Course Overview for more details)</td></tr>
<tr><td>G.GSR.4.1</td><td>Use the undefined notions of point, line, line segment, plane, distance along a line segment, and distance around a circular arc to develop and use precise definitions and symbolic notations to prove theorems and solve geometric problems.</td><td colspan="2">Fundamentals
• Student should be provided opportunities to build a conceptual understanding of a point, line, line segment, plane, arc, and angle through modeling and exploration of real-life phenomena.
• Students should attend to precision when using definitions and symbolic notations.
• Students should be able to apply the Segment Addition Postulate and Angle Addition Postulate to solve real-life problems.
• Students should read, write, use, and interpret symbolic notation for point, line, plane, line segment, angle, circle, arc, perpendicular line, and parallel line.</td></tr>
<tr><td>G.GSR.4.2</td><td>Classify quadrilaterals in the coordinate plane by proving simple geometric theorems algebraically.</td><td colspan="2">Fundamentals
• Students should build on their existing understanding of the slope of a line segment developed in the Algebra: Concepts and Connections course.
• Students should be able to classify quadrilaterals as parallelograms (including rectangles, rhombi, and squares) using sides, angles, and diagonals.
• Students should be able to apply their understanding of slope of a line segment, as well as distance and midpoint formulas to classify quadrilaterals in the coordinate plane.</td></tr>
<tr><td>G.GSR.4.3</td><td>Make formal geometric constructions with a variety of tools and methods.</td><td>Strategies and Methods
• Students should have opportunities to use a variety of tools, which might include a compass and straightedge, string, reflective</td><td>Fundamentals
• Student should be able to: Copy a segment and angle.
   ○ Bisect a segment and angle.</td></tr>
</table>

		devices, paper folding, dynamic geometric software, etc.	○ Construct perpendicular lines, including the perpendicular bisector of a line segment. ○ Construct a line parallel to a given line through a point not on the line.	
G.GSR.4.4	Prove and apply theorems about lines and angles to solve problems.	***Fundamentals*** • Students should be given opportunities to precisely prove vertical angles are congruent. • Students should be given opportunities to explore using visual tools in order to precisely prove when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent. • Students should be provided with opportunities to analyze and apply theorems about lines and angles from the context of parallel lines cut by a transversal to make sense of relationships between lines and angles. • Students should be given opportunities to precisely prove that points on the perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. • Students should be able to show and explain their reasoning used to generate their proof.	***Relevance and Application*** • Students should be able to apply theorems to solve problems and to prove relationships in geometric figures by applying geometric and algebraic reasoning.	
G.GSR.4.5	Use geometric reasoning to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.	***Strategies and Methods*** ● Use informal (visual) construction with tools (patty paper, protractor, etc.) to discover the angle relationships between angles formed when two lines are cut by a transversal. ● When using more than one transversal, tie into similar triangles and to set up problems using triangle sum relationships (angle sum).	***Terminology*** • Including identify alternate exterior angles, alternate interior angles, linear pairs, same side interior angles, same side exterior angles, and corresponding angles.	***Example*** • For example, arrange three copies of the same triangle so that the three angles appear to form a line, and give an argument in terms of transversals why this is so.

GEOMETRIC & SPATIAL REASONING – *similarity*

G.GSR.5: Describe dilations in terms of center and scale factor and use these terms to describe properties of dilations; use the precise definition of a dilation to describe similarity and establish the criterion for triangles to be similar; use these terms, definitions, and criterion to prove similarity, model, and explain real-life phenomena.

	Expectations	Evidence of Student Learning (not all inclusive; see Course Overview for more details)		
G.GSR.5.1	Verify experimentally the properties of dilations.	***Fundamentals*** • Students should be able to identify dilation as reduction or enlargement depending on scale factor. • Students should be given multiple opportunities to draw a dilated image given the center at the origin and scale factor. • Students should be able to describe a dilation by identifying its center through the intersection of lines going through corresponding preimage and image points by finding the ratio of sides of the image to the preimage as its scale factor. • Students should be able to understand and use function notation to represent dilations in the coordinate plane. • Students should be able to describe properties of dilations, such as center, scale factor, angle measure, parallelism, and collinearity.	***Strategies and Methods*** • Dilations should be limited to those centered at the origin.	***Example*** • The function notation (x, y) --> (4x, 4y) enlarges the point (x, y) with a scale factor of four.
G.GSR.5.2	Given two figures, use and apply the definition of similarity in terms of similarity transformations.	***Fundamentals*** • Students should be able to explain using similarity transformations the meaning of similarity for figures as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. • Students should apply properties of similarity to solve problems with missing values involving corresponding parts.		
G.GSR.5.3	Use the properties of similarity transformations to establish criterion for two triangles to be similar. Use similarity criteria	***Fundamentals*** • Students should be able to apply properties of similarity to solve problems with missing values involving corresponding parts.	***Strategies and Methods*** • Students should be given opportunities to explore the AA, SAS, and SSS similarity	

	for triangles to solve problems and to prove relationships in geometric figures.		postulates/theorems and use these to prove triangles are similar. • Students should be able to prove that two triangles are similar using appropriate methods (logic statements, paragraph proofs, two-column proofs, or flowchart proofs).
G.GSR.5.4	Construct formal proofs to justify and apply theorems about triangles.	***Fundamentals*** • Students should be able to prove a line parallel to one side of a triangle divides the other two proportionally, and its converse. • Students should be able to prove the Pythagorean Theorem using triangle similarity.	***Relevance and Application*** • Students should be able to apply these theorems, as well as the Midsegment and Angle Bisector Theorems to solve problems in similar geometric figures.

GEOMETRIC & SPATIAL REASONING – *right triangle trigonometry*		
G.GSR.6: Examine side ratios of similar triangles; use the relationship between right triangles to develop an understanding of sine, cosine, and tangent to solve mathematically applicable geometric problems and to model and explain real-life phenomena.		
	Expectations	**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
G.GSR.6.1	Explain that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	***Fundamentals*** • Students should be able to use similarity to establish sine, cosine, and tangent ratios.
G.GSR.6.2	Explain and use the relationship between the sine and cosine of complementary angles.	***Fundamentals*** • Students should be able to verify and apply the relationship between cofunctions, $\sin(\theta) = \cos(90° - \theta)$ and $\cos(\theta) = \sin(90° - \theta)$. • In seventh grade, students write and solve equations using supplementary, complementary, vertical, and adjacent angles.
G.GSR.6.3	Use trigonometric ratios and the Pythagorean Theorem to solve for sides and angles of right triangles in applied problems.	***Strategies and Methods*** • Students should be able to use sine, cosine, and tangent to solve real-life problems that require them to find missing side and angle measurements.

GEOMETRIC & SPATIAL REASONING – *Trigonometry and the Unit Circle*		
G.GSR.7: Explore the concept of a radian measure and special right triangles.		
	Expectations	**Evidence of Student Learning**
G.GSR.7.1	Explore and interpret a radian as the ratio of the arc length to the radius of a circle.	***Strategies and Methods*** • Students should be given opportunities to make sense of the meaning of radians conceptually through exploration with visual tools. • Using hands on tools and technology visualizations, students should have opportunities to explore and develop an understanding of the relationship between the radius of a circle, an arc length, and the associated radian measure.
G.GSR.7.2	Explore and explain the relationship between radian measures and degree measures and convert fluently between degree and radian measures.	***Fundamentals*** • Students should be able to convert fluently (flexibly, accurately, and efficiently) between degree and radian measures to solve real-life problems. ***Strategies and Methods*** • Students should have opportunities to explore and discover experimentally the relationship between radian measure and degree measure using visual tools.
G.GSR.7.3	Use special right triangles on the unit circle to determine the values of sine, cosine, and tangent for 30° ($\frac{\pi}{6}$), 45° ($\frac{\pi}{4}$) and 60° ($\frac{\pi}{3}$) angle measures. Use reflections of triangles to determine reference angles and identify coordinate values in all four quadrants of the coordinate plane.	***Fundamentals*** • Students should be able to articulate the pattern associated with angle measures in all four quadrants of the unit circle, e.g., 150° as 180°-30°, 210° as 180°+30°, 330° as 360°-30°, etc. • Students should explore, interpret, and use radian measures based on conversions from degree measures, such as 150°, 210°, etc., and articulate the patterns associates with those radian measures, including the connection of $\frac{5\pi}{6} \approx 2.617$ radius units measured along the arc length of the circle. • Through explorations, students develop an understanding that a unit circle has a radius equal to 1. • This learning objective is limited to angle measures of 30° ($\frac{\pi}{6}$), 45° ($\frac{\pi}{4}$) and 60° ($\frac{\pi}{3}$), and their associated reflected angles within one counterclockwise revolution of the unit circle.

GEOMETRIC & SPATIAL REASONING *– circles*		
G.GSR.8: Examine and apply theorems involving circles; describe and derive arc length and area of a sector; and model and explain real-life frameworks involving circles.		
	Expectations	**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
G.GSR.8.1	Identify and apply angle relationships formed by chords, tangents, secants and radii with circles.	***Fundamentals*** • Real-life frameworks should include: ○ angles based on the location of the vertex: central, inscribed, interior, and exterior. ○ the angle formed at the intersection of the radius of a circle and a segment tangent to the circle (point of tangency); determining these segments are perpendicular. ○ triangles inscribed in and circumscribed about circles. ○ opposite angles of a quadrilateral inscribed in a circle; determining these angles are supplementary.
G.GSR.8.2	Using similarity, derive the fact that the length of the arc (arc length) intercepted by an angle is proportional to the radius; derive the formula for the area of a sector. Solve mathematically applicable problems involving applications of arc length and area of sector.	***Fundamentals*** • Students should be able to apply strategic thinking and complex reasoning when solving problems involving arc length and area of a sector of a circle. ***Strategies and Methods*** • Students should be given opportunities to use interactive tools to engage with the content in order to develop a conceptual understanding of arc length and area of a sector.
G.GSR.8.3	Write and graph the equation of circles in standard form.	***Terminology*** • The general form of the equation for a circle is $x^2 + y^2 + Cx + Dy + E = 0$. • The standard form of the equation for a circle is $(x-h)^2 + (y-k)^2 = r^2$. ***Fundamentals*** • Students should be able to identify the center and radius of a circle from an equation in standard form or from the graph of a circle. • Students should be able to write the equation of a circle in standard form given the graph of the circle. • Students should be able to graph a circle from the standard form equation of a circle. • As students convert equations in general form to standard form in this course, the leading coefficient of the quadratic terms should be limited to 1. ***Strategies and Methods*** • Students may use a variety of methods to convert the equation of a circle in general form to the equation of a circle in standard form for a specific, circumstantial purpose. One strategy used by students may include (but is not limited to) completing the square.

GEOMETRIC & SPATIAL REASONING – equations and measurement			
G.GSR.9: Develop informal arguments for geometric formulas using dissection arguments, limit arguments, and Cavalieri's principle; solve mathematically applicable problems involving volume; explore and visualize relationships between two-dimensional and three-dimensional objects to model and explain real-life phenomena.			
	Expectations	**Evidence of Student Learning** (not all inclusive; see Course Overview for more details)	
G.GSR.9.1	Use volume formulas for prisms, cylinders, pyramids, cones, and spheres to solve problems including right and oblique solids.	***Fundamentals*** • Students should be able to verify experimentally the formulas for the volume of a cylinder, pyramid, sphere, prism and cone; emphasize volume as the product of the area of the base and the height for both prisms and cylinders. • Students should be able to use and explain Cavalieri's Principle to show that the volume of an oblique solid can be found using right solids. • Students should find the volume of solids and composite solids to explain real-life phenomena.	***Terminology*** • Prism – a solid figure that has the same cross section all along its length
G.GSR.9.2	Use geometric shapes, their measures, and their properties to describe objects and approximate volumes.	***Strategies and Methods*** • Students should be able to choose the appropriate geometric solid to approximate volumes of irregular objects.	***Example*** • Modeling a tree trunk or a human torso as a cylinder
G.GSR.9.3	Apply concepts of density based on area and volume in modeling situations	***Strategies and Methods*** • Students should be able to choose the appropriate geometric figure or solid to approximate density of irregular objects in a geometric situation.	***Example*** • Persons per square mile, fish per cubic feet of a fish tank

PROBABILISTIC REASONING – compound events and expected values			
G.PR.10: Solve problems involving the probability of compound events to make informed decisions; interpret expected value and measures of variability to analyze probability distributions.			
	Expectations	**Evidence of Student Learning**	
G.PR.10.1	Describe categories of events as subsets of a sample space using unions, intersections, or complements of other events. Apply the Addition Rule conceptually, P(A or B)= P(A) + P(B)-P(A and B), and interpret the answers in context.	***Fundamentals*** • Students should be able to communicate informed decisions by applying the Addition Rule to a problem involving the probability of compound events. • The focus and emphasis should be on the understanding of the Addition Rule conceptually with limited emphasis on the manipulation of the equation.	***Strategies and Methods*** • Students should have opportunities using various tools such as Venn Diagrams and two-way tables to help visualize events. • Two-way tables can be used to reveal all the sample space. Venn diagrams can be used to show intersections of two or more variables.

G.PR.10.2	Apply and interpret the general Multiplication Rule conceptually to independent events of a sample space, P(A and B) = [P(A)]x[P(B|A)] =[P(B)]x[P(A|B)] using contingency tables or tree diagrams.	***Fundamentals*** • Students should be able to relate the conditional probability back to the conceptual interpretation of probability studied in previous courses. • The focus and emphasis should be on the understanding of the Multiplication Rule conceptually with limited emphasis on the manipulation of the equation.	***Strategies and Methods*** • Tree diagrams may be used to help students visualize events and probabilities of those events.
G.PR.10.3	Use conditional probability to interpret risk in terms of decision-making and investigate questions such as those involving false positives or false negatives from screening tests.	***Fundamentals*** • Relevant questions should be answered based on the appropriate risk measures. • Students should be able to explain how studies and/or models were used to determine the risk measures. • Students should be able to recognize that the chances of a false positive or a false negative are not the same as the chances of having the condition or not having the condition given the test result. • Students should be able to interpret and communicate the consequences, of making the false positive or false negative errors. • Students should be able to interpret the notation for conditional probability in context.	***Terminology*** • A false positive is the probability of a positive result given the condition is not present. • A false negative is the probability of a negative result given the condition is present. ***Examples*** • Given a positive test result, what are the chances the person has the illness measured in the screening test? • Given that a person has the illness, what are the chances of them getting a positive test result on the screening test.
G.PR.10.4	Define permutations and combinations and apply this understanding to compute probabilities of compound events and solve meaningful problems.	*Fundamentals* • Students should understand the terms permutation and combination and be able to solve simple problems involving selection and arrangements of objects in a line, including those involving repetition and restriction. • The emphasis should be on the conceptual understanding and application of ***Terminology*** • A permutation is a special case of an arrangement. • A combination is a special case of a selection. • Repetition is a type of permutation where a repeat of elements from the set is allowed. • Restriction is a type of permutation where each element is used only once, and a certain order is required.	***Examples*** • The Georgia Department of Transportation (GDOT) is creating new license plates. How many plates can they create using the symbols L, M, T, O, 3, 4. Students should recognize this problem as an example of a permutation and be able to determine the number of unique license plates that can be produced. • There are five students in a group that need to form a

		combinations and permutations. • Students should be able to use and interpret formal notation to communicate about combinations and permutations (e.g., $_nP_r$ and $_nC_r$ to represent choosing *r* objects from *n* distinct objects).		straight line. Students A and B cannot stand next to each other. How many ways can they stand in line?
G.PR.10.5	Interpret the probability distribution for a given random variable and interpret the expected value.	***Fundamentals*** • Students should be able to understand that the probabilities in a distribution are between 0 and 1, and that they should sum to 1. • Students should define random variable and understand that the sample space consists of all the values the random variable can take. • Through mathematically applicable explorations, students should develop an understanding that the expected value is the mean of the probability distribution. • Students should be presented with culturally relevant problems where they are given the expected value and can interpret its meaning within context.		
G.PR.10.6	Develop a probability distribution for variables of interest using theoretical and empirical (observed) probabilities and calculate and interpret the expected value.	***Fundamentals*** • Students should be able to calculate the probability of all possible outcomes of a given event and display the probability of each graphically. • Students should understand that the sum of all the probabilities within one distribution will be 1 (100%).	***Strategies and Methods*** • A chart showing every outcome and the resulting probabilities might be useful in graphing the probability distribution. • Utilizing notation X as a discrete random variable denoting an outcome, P(X) is the probability the outcome occurs. • Students should be able to find the probability of a certain quantity (e.g., P(X = 2)), and also the probability of a range of quantities (e.g., P(X > 2)).	***Example*** • Define X as the number of “tails” we get after three flips of a fair coin, students should first realize that in 3 flips, they could get X=0 tails, X=1 tail, X=2 tails, or X=3 tails. Using the sample space, (HHH, HHT, HTH, HTT, THH, THT, TTH, TTT) students calculate P(X), the probability of each X value, above. P(0) = 0.125, P(1)=0.375, P(2)=0.375, and P(3)=0.125. Students would show this distribution graphically.
G.PR.10.7	Calculate the expected value of a random variable and interpret it as the mean of a given probability distribution.	***Fundamentals*** • Students should be able to use the expected value of a random variable to make informed decisions.		***Example*** • Using the probability distribution that represents the number of tails you flip in three flips of a coin, the probability distribution would be

		• Students should calculate the expected value of a random variable as the sum of each X_n * $P(X_n)$, and understand that this sum is the weighted average of the outcomes (weighted by the probability).	(0)(0.125)+(1)*(0.375)+(2)(0.375)+(3)*(0.125)=1.5. So, on average, in three flips of the fair coin, you will get 1.5 tails. Students should realize that it is not possible to get 1.5 tails, and that 1.5 is exactly halfway between 1 and 2, and therefore it is just as likely to get 1 tail in 3 flips as it is to get 2 tails.
G.PR.10.8	Compare the payoff values associated with the probability distribution for a random variable and make informed decisions based on expected value and measures of variability.	***Fundamentals*** • Students should consider net value or payoff when making decisions about real-life problems. • Students should understand that two probability distributions can have the same expected value, but one may vary more than the other, and this should be considered in decision-making. • It is not necessary to calculate the standard deviation of the probability distribution.	***Examples*** • Students can compute and interpret expected values for games of chance, insurance policies, and other real-life situations.

DATA & STATISTICAL REASONING; PROBABLISTIC REASONING – *categorical data in two-way frequency tables; conditional probability*			
G.DSR.11: Examine real-life situations presented in a two-way frequency table to calculate probabilities, to model categorical data, and to explain real-life phenomena.			
	Expectations	**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**	
G.DSR.11.1	Construct and summarize categorical data for two categories in two-way frequency tables.	***Fundamentals*** • Students should be able to identify, calculate, and interpret joint, marginal, and conditional relative frequencies in context of the data. • Students should have opportunities to analyze meaningful, real-life data and recognize possible associations and trends in the data. • Students should understand and apply concepts of sample space to describe categorical data.	
G.DSR.11.2	Use categorical data in two-way frequency tables to calculate and interpret probabilities based on the investigation.	***Terminology*** • Respective symbolic notation: P(A and B) = $P(A \cap B)$ and P(A or B) = $P(A \cup B)$.	***Fundamentals*** • Students should be able to use two-way frequency tables to find probabilities for unions and intersections. • Students should have opportunities to use two-way frequency tables to compute conditional probabilities.

ESSENTIAL INSTRUCTIONAL GUIDANCE

MATHEMATICAL PRACTICES

The Mathematical Practices describe the reasoning behaviors students should develop as they build an understanding of mathematics – the “habits of mind” that help students become mathematical thinkers. There are eight standards, which apply to all grade levels and conceptual categories.

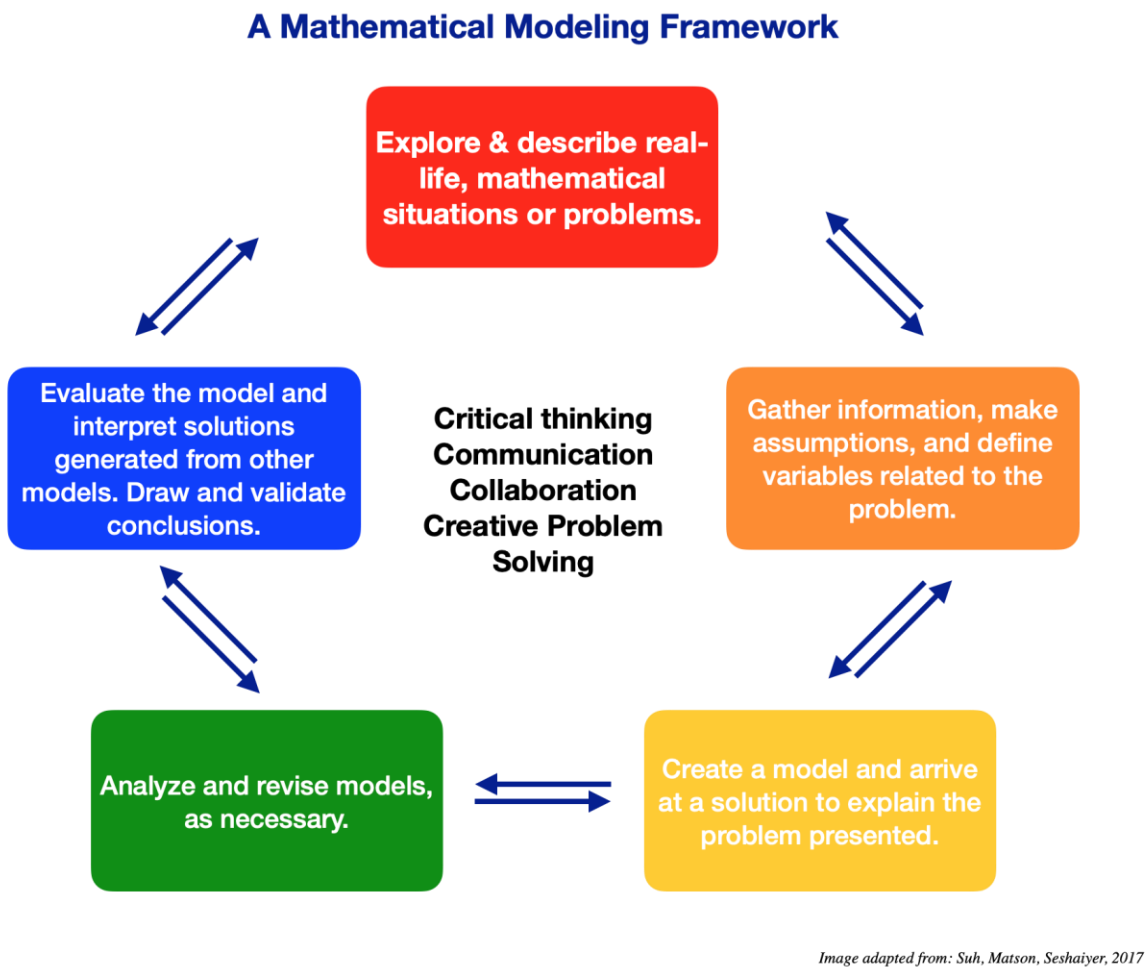
These mathematical practices describe how students should engage with the mathematics content for their grade level. Developing these habits of mind builds students’ capacity to become mathematical thinkers. These practices can be applied individually or together in mathematics lessons, and no particular order is required. In well-designed lessons, there are often two or more Standards for Mathematical Practice present.

Mathematical Practices	
G.MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.	
Code	**Expectation**
G.MP.1	Make sense of problems and persevere in solving them.
G.MP.2	Reason abstractly and quantitatively.
G.MP.3	Construct viable arguments and critique the reasoning of others.
G.MP.4	Model with mathematics.
G.MP.5	Use appropriate tools strategically.
G.MP.6	Attend to precision.
G.MP.7	Look for and make use of structure.
G.MP.8	Look for and express regularity in repeated reasoning.

MATHEMATICAL MODELING

Teaching students to model with mathematics is engaging, builds confidence and competence, and gives students the opportunity to collaborate and make sense of the world around them, the main reason for doing mathematics. For these reasons, mathematical modeling should be incorporated at every level of a student's education. This is important not only to develop a deep understanding of mathematics itself, but more importantly to give students the tools they need to make sense of the world around them. Students who engage in mathematical modeling will not only be prepared for their chosen career but will also learn to make informed daily life decisions based on data and the models they create.

The diagram below is a mathematical modeling framework depicting a cycle of how students can engage in mathematical modeling when solving a real-life problem or task.



FRAMEWORK FOR STATISTICAL REASONING

Statistical reasoning is important for learners to engage as citizens and professionals in a world that continues to change and evolve. Humans are naturally curious beings and statistics is a language that can be used to better answer questions about personal choices and/or make sense of naturally occurring phenomena. Statistics is a way to ask questions, explore, and make sense of the world around us.

The Framework for Statistical Reasoning should be used in all grade levels and courses to guide learners through the sense-making process, ultimately leading to the goal of statistical literacy in all grade levels and courses. Reasoning with statistics provides a context that necessitates the learning and application of a variety of mathematical concepts.

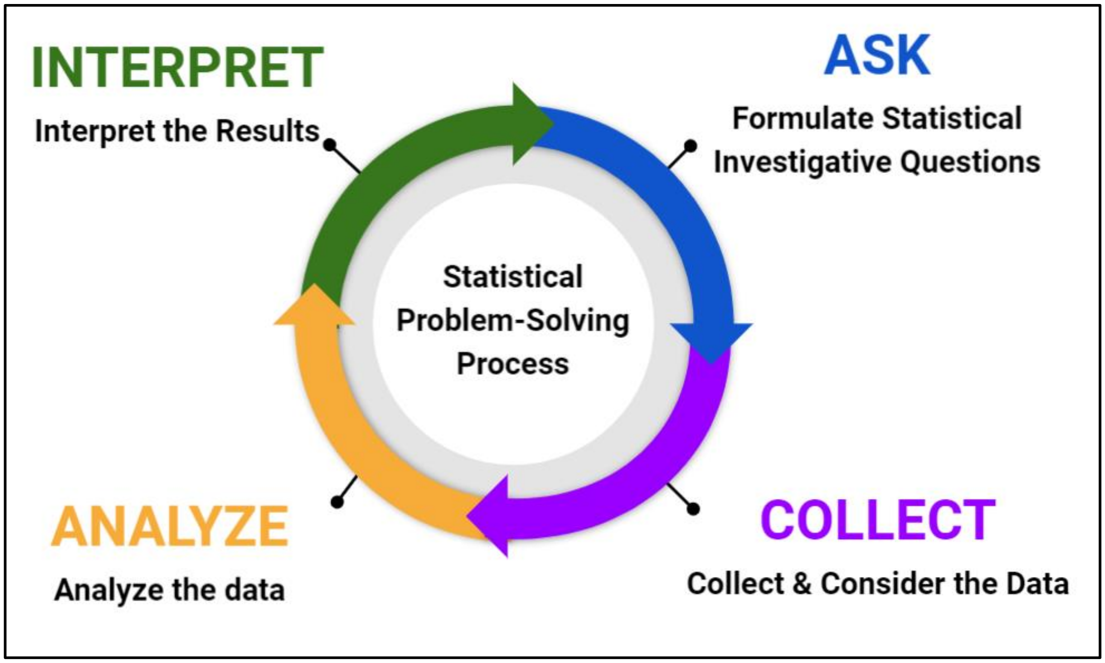


Figure 1: Georgia Framework for Statistical Reasoning

The following four-step statistical problem-solving process can be used throughout each grade level and course to help learners develop a solid foundation in statistical reasoning and literacy:

I. Formulate Statistical Investigative Questions
Ask questions that anticipate variability.

II. Collect & Consider the Data
Ensure that data collection designs acknowledge variability.

III. Analyze the Data
Make sense of data and communicate what the data mean using pictures (graphs) and words. Give an accounting of variability, as appropriate.

IV. Interpret the Results
Answer statistical investigative questions based on the collected data.