Harrison High School
Multivariable Calculus Prerequisite Packet

To: Multivariable Students and Parents
From: Monica Doriney, Calculus Instructor

Multivariable Calculus is a fourth-year mathematics course option for students who have completed AP Calculus BC. It includes three-dimensional coordinate geometry; matrices and determinants; eigenvalues and eigenvectors of matrices; limits and continuity of functions with two independent variables; partial differentiation; multiple integration; the gradient; the divergence; the curl; Theorems of Green, Stokes, and Gauss; line integrals; integrals independent of path; and linear first-order differential equations.

The Prerequisite Packet: Students need a strong foundation to be ready for the rigorous work required throughout the term. Completing the prerequisite packet should prepare you for the material to be taught in the course. This packet consists of material studied during Algebra II, Pre-calculus, and Calculus. Students should anticipate working approximately 4 hours to complete it properly. The packet will be collected on the first day of class. Do not list only an answer. Work neatly and in an organized fashion.

I anticipate a motivating and challenging year. Multivariable Calculus is a stimulating and exciting field of mathematics and I look forward to sharing my excitement with you. I will be there to help and support you.

Prerequisite Problems

Trigonometric Identities

1. Verify the Identity:
$$5\cos^4\theta - 5\sin^4\theta = 5\cos 2\theta$$

2. Verify the Identity:
$$\frac{\csc^2 x + 2\csc x - 3}{\csc^2 x - 1} = \frac{\csc x + 3}{\csc x + 1}$$

3. Verify the Identity:
$$2\cos^2\left(\frac{x}{4}\right) = 1 + \cos\left(\frac{x}{2}\right)$$

4. Verify the Identity:
$$\frac{\sin(x+h) - \sin x}{h} = \sin x\left(\frac{\cos(h) - 1}{h}\right) + \cos x\left(\frac{\sin(h)}{h}\right)$$

Conics

5. Write standard form, the graph and identify the conic.
$$x^2 + 4x + 4y + 24 = 0$$

6. Write standard form, the graph and identify the conic.
$$x^2 - 4y^2 - 6x - 16y - 11 = 0$$

7. Write standard form, the graph and identify the conic.

$$x^2 - 2x + y^2 - 4y - 25 = 0$$

8. Write standard form, the graph and identify the conic.

$$4x^2 + 24x + 25y^2 - 300y + 836 = 0$$

Integration

9. $\int_1^4 \frac{1}{\sqrt{x}\left(\sqrt{x}+1\right)^2} dx$

10. $\int \sin x \cdot \sec x \; dx$

11. $\int x \cdot \tan^2 x \, dx$

12. $\int \sec^3 x \cdot \tan^3 x \; dx$

13. $\int x^2 \tan^{-1} x \ dx$

14. $\int \frac{\sec^2 x}{(1+\tan x)^3} dx$

15. $\int \sin^4 \theta \ d\theta$

16. $\int_{-1}^{1} x^2 \sqrt{x^3 + 1}\, dx$

Differentiation

17. $y = (x^4 + \sin x \cdot \cos x)^3$

18. $y = \frac{\tan x}{x^2+2}$

19. $y = \sqrt{\frac{1+x}{2-x}}$

20. $y = \arctan(3x - 1) + \sqrt{\sin(\ln x)}$

21. $y = \cos^2\left(x^2 + \frac{x}{x+1}\right)$

22. $\text{y} = \ln\left(\frac{2(1+x^2)}{x^4}\right)$

Implicit Differentiation

23. Find $\frac{dy}{dx}$

$$x^3 - 3x^2y + 4xy^2 = 12$$

24. Find $\frac{dy}{dx}$

$$4 \sin 2y \cos x = 2$$

25. Find $\frac{dy}{dx}$

$$(y^2 + 2\sec y)^2 = 4(x+1)^2$$

26. Find $\frac{dy}{dx}$

$$x = y \cdot \sec\left(\frac{5}{y}\right)$$

Parametric Equations

27. Given the parametric equations $x = 2\sqrt{t}$ and $y = 3t^2 - 2t$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

28. Given the parametric equations $x = 4\cos t$ and $y = 3\sin t$, write an equation of the tangent line to the curve at the point where $t = \frac{3\pi}{4}$.

29. Find all points of horizontal and vertical tangency given the parametric equations $x = t^2 + t$ and $y = t^2 - 3t + 5$

30. Set up an integral expression for the arc length of the curve given by the parametric equations $x = t^2 + 1$, $y = 4t^3 - 1$, $0 \le t \le 1$. Do not evaluate.

Polar

31. Given $r = 3 + 4\cos\theta$, find $\frac{dy}{dx}$ and the slope of the tangent line to the polar curve at $\theta = \frac{\pi}{2}$.

32. Find the area inside the circle $r = 4$ and outside the cardioid $r = 2 - 2\cos\theta$.

33. Find the arc length of the spiral $r = e^{2\theta}$ between $\theta = 0$ and $\theta = \frac{\pi}{2}$.

34. Find the values θ where the equation $r = 3\cos\theta$ has (a) vertical tangent(s). $0 \le \theta \le 2\pi$

Vectors

35. What is the maximum acceleration attained on the interval $0 \le t \le 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$

36. The position of a particle moving along a line is given by $s(t) = 2t^3 - 24t^2 + 90t + 7$ for $t \ge 0$. For what values of t is the speed of the particle increasing?

37. A particle moves along the x-axis so that at any time $t > 0$, its acceleration is given by $a(t) = \ln(1+2^t)$. If the velocity of the particle is 2 at time $t = 1$, then what is the velocity of the particle at time t = 2.

38. A particle travels along a straight line with a velocity of $v(t) = 3e^{-\frac{t}{2}}\sin(2t)$ meters per second. What is the total distance traveled by the particle during the time interval $0 \leq t \leq 2$ seconds?

Taylor Series

39. The Taylor Series of a function $f(x)$ about $x = 3$ is given by
$f(x) = 1 + \frac{3(x-3)}{1!} + \frac{5(x-3)^2}{2!} + \frac{7(x-3)^3}{3!} + \cdots + \frac{(2n+1)(x-3)^n}{n!} + \cdots$
What is the value of $f'''(3)$?

40. What is the sum of the Maclaurin se
$$\pi - \frac{\pi^3}{3!} + \frac{\pi^5}{5!} - \frac{\pi^7}{7!} + \ldots + (-1)^n \frac{\pi^{2n+1}}{(2n+1)!} + \ldots$$