CollegeBoard

The SAT®
Mathematics Review

achieve more™

SAT®

Mathematics Review

Number and Operations (20–25%)

- Arithmetic word problems (including percent, ratio, and proportion)
- Properties of integers (even, odd, prime numbers, divisibility, etc.)
- Rational numbers
- Sets (union, intersection, elements)
- Counting techniques
- Sequences and series (including exponential growth)
- Elementary number theory

Algebra and Functions (35–40%)

- Substitution and simplifying algebraic expressions
- Properties of exponents
- Algebraic word problems
- Solutions of linear equations and inequalities
- Systems of equations and inequalities
- Quadratic equations
- Rational and radical equations
- Equations of lines
- Absolute value
- Direct and inverse variation
- Concepts of algebraic functions
- Newly defined symbols based on commonly used operations

Geometry and Measurement (25–30%)

- Area and perimeter of a polygon
- Area and circumference of a circle
- Volume of a box, cube, and cylinder
- Pythagorean Theorem and special properties of isosceles, equilateral, and right triangles
- Properties of parallel and perpendicular lines
- Coordinate geometry
- Geometric visualization
- Slope
- Similarity
- Transformations

Data Analysis, Statistics, and Probability (10–15%)

- Data interpretation (tables and graphs)
- Descriptive statistics (mean, median, and mode)
- Probability

Number and Operations

- **Integers:** $\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots$
  (Note: zero is neither positive nor negative.)
- **Consecutive Integers:** Integers that follow in sequence; for example, 22, 23, 24, 25. Consecutive integers can be more generally represented by $n, n+1, n+2, n+3, \ldots$
- **Odd Integers:** $\ldots, -7, -5, -3, -1, 1, 3, 5, 7, \ldots, 2k+1, \ldots$ where k is an integer
- **Even Integers:** $\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots, 2k, \ldots$, where k is an integer (*Note: zero is an even integer.)*
- **Prime Numbers:** 2, 3, 5, 7, 11, 13, 17, 19, . . .
  (Note: 1 is not a prime and 2 is the only even prime.)
- **Digits:** 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
  (Note: the units digit and the ones digit refer to the same digit in a number. For example, in the number 125, the 5 is called the units digit or the ones digit.)

Percent

Percent means hundredths, or number out of 100. For example, 40 percent means $\frac{40}{100}$ or 0.40 or $\frac{2}{5}$.

Problem 1: If the sales tax on a \$30.00 item is \$1.80, what is the sales tax rate?

Solution: $\$1.80 = \frac{n}{100} \times \$30.00$

$n = 6$, so 6% is the sales tax rate.

Percent Increase / Decrease

Problem 2: If the price of a computer was decreased from \$1,000 to \$750, by what percent was the price decreased?

Solution: The price decrease is \$250. The percent decrease is the value of n in the equation $\frac{250}{1{,}000} = \frac{n}{100}$. The value of n is 25, so the price was decreased by 25%.

Note: n% increase means $\frac{\text{increase}}{\text{original}} = \frac{n}{100}$;

n% decrease means $\frac{\text{decrease}}{\text{original}} = \frac{n}{100}$.

Average Speed

Problem: José traveled for 2 hours at a rate of 70 kilometers per hour and for 5 hours at a rate of 60 kilometers per hour. What was his average speed for the 7-hour period?

Solution: In this situation, the average speed was

$$\frac{\text{total distance}}{\text{total time}}$$

The total distance was

$$2\text{ hr}\left(70\frac{\text{km}}{\text{hr}}\right) + 5\text{ hr}\left(60\frac{\text{km}}{\text{hr}}\right) = 440\text{ km}.$$

The total time was 7 hours. Thus, the average speed was

$$\frac{440\text{ km}}{7\text{ hr}} = 62\frac{6}{7}\text{ kilometers per hour.}$$

Note: In this example, the average speed over the 7-hour period is not the average of the two given speeds, which would be 65 kilometers per hour.

Sequences

Two common types of sequences that appear on the SAT are arithmetic and geometric sequences.

An **arithmetic sequence** is a sequence in which successive terms differ by the same constant amount.

For example: 3, 5, 7, 9, . . . is an arithmetic sequence.

A **geometric sequence** is a sequence in which the ratio of successive terms is a constant.

For example: 2, 4, 8, 16, . . . is a geometric sequence.

A sequence may also be defined using previously defined terms. For example, the first term of a sequence is 2, and each successive term is 1 less than twice the preceding term. This sequence would be 2, 3, 5, 9, 17, . . .

On the SAT, explicit rules are given for each sequence. For example, in the sequence above, you would not be expected to know that the 6th term is 33 without being given the fact that each term is one less than twice the preceding term. For sequences on the SAT, the first term is *never* referred to as the zeroth term.

Algebra and Functions

Factoring

You may need to apply these types of factoring:

$$x^2 + 2x = x(x+2)$$

$$x^2 - 1 = (x+1)(x-1)$$

$$x^2 + 2x + 1 = (x+1)(x+1) = (x+1)^2$$

$$2x^2 + 5x - 3 = (2x-1)(x+3)$$

Functions

A function is a relation in which each element of the domain is paired with *exactly* one element of the range. On the SAT, unless otherwise specified, the domain of any function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. For example, if $f(x) = \sqrt{x+2}$, the domain of f is all real numbers greater than or equal to -2. For this function, 14 is paired with 4, since $f(14) = \sqrt{14+2} = \sqrt{16} = 4$.

Note: the $\sqrt{\ }$ symbol represents the positive, or principal, square root. For example, $\sqrt{16} = 4$, not ± 4.

Exponents

You should be familiar with the following rules for exponents on the SAT.

For all values of a, b, x, y:

$$x^a \cdot x^b = x^{a+b} \qquad \left(x^a\right)^b = x^{a \cdot b} \qquad (xy)^a = x^a \cdot y^a$$

For all values of $a,\ b,\ x > 0,\ y > 0$:

$$\frac{x^a}{x^b} = x^{a-b} \qquad \left(\frac{x}{y}\right)^a = \frac{x^a}{y^a} \qquad x^{-a} = \frac{1}{x^a}$$

Also, $x^{\frac{a}{b}} = \sqrt[b]{x^a}$. For example, $x^{\frac{2}{3}} = \sqrt[3]{x^2}$.

Note: For any nonzero number x, it is true that $x^0 = 1$.

Variation

Direct Variation: The variable y is directly proportional to the variable x if there exists a nonzero constant k such that $y = kx$.

Inverse Variation: The variable y is inversely proportional to the variable x if there exists a nonzero constant k such that $y = \frac{k}{x}$ or $xy = k$.

Absolute Value

The absolute value of x is defined as the distance from x to zero on the number line. The absolute value of x is written as $|x|$. For all real numbers x:

$$|x| = \begin{cases} x, & \text{if } x \ge 0 \\ -x, & \text{if } x < 0 \end{cases}$$

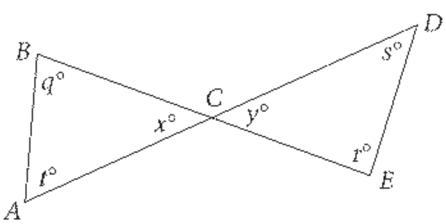
For example: $\begin{cases} |2| = 2, & \text{since } 2 > 0 \\ |-2| = -(-2) = 2, & \text{since } -2 < 0 \\ |0| = 0 \end{cases}$

Geometry and Measurement

Figures that accompany problems are intended to provide information useful in solving the problems. They are drawn as accurately as possible EXCEPT when it is stated in a particular problem that the figure is not drawn to scale. In general, even when figures are not drawn to scale, the relative positions of points and angles may be assumed to be in the order shown. Also, line segments that extend through points and appear to lie on the same line may be assumed to be on the same line. A point that appears to lie on a line or curve may be assumed to lie on the line or curve.

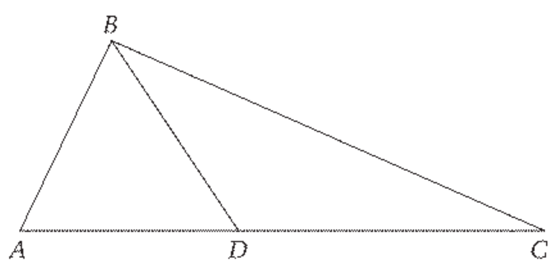
The text "Note: Figure not drawn to scale" is included with the figure when degree measures may not be accurately shown and specific lengths may not be drawn proportionally. The following examples illustrate what information can and cannot be assumed from figures.

Example 1:



Since $\overline{AD}$ and $\overline{BE}$ are line segments, angles *ACB* and *DCE* are vertical angles. Therefore, you can conclude that $x = y$. Even though the figure is drawn to scale, you should NOT make any other assumptions without additional information. For example, you should NOT assume that $AC = CD$ or that the angle at vertex *E* is a right angle even though they might look that way in the figure.

Example 2:



Note: Figure not drawn to scale.

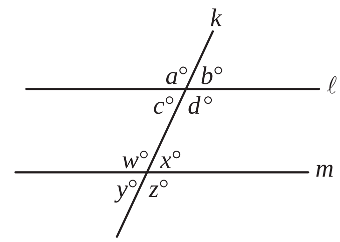
A question may refer to a triangle such as *ABC* above. Although the note indicates that the figure is not drawn to scale, you may assume the following from the figure:

- *ABD* and *DBC* are triangles.
- *D* is between *A* and *C*.
- *A*, *D*, and *C* are points on a line.
- The length of $\overline{AD}$ is less than the length of $\overline{AC}$.
- The measure of angle *ABD* is less than the measure of angle *ABC*.

You may *not* assume the following from the figure:

- The length of $\overline{AD}$ is less than the length of $\overline{DC}$.
- The measures of angles *BAD* and *BDA* are equal.
- The measure of angle *ABD* is greater than the measure of angle *DBC*.
- Angle *ABC* is a right angle.

Properties of Parallel Lines



1. If two parallel lines are cut by a third line, the alternate interior angles are congruent. In the figure above,

$$c = x \text{ and } w = d.$$

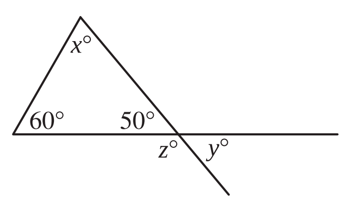
2. If two parallel lines are cut by a third line, the corresponding angles are congruent. In the figure,

$$a = w, b = x, c = y, \text{ and } d = z.$$

3. If two parallel lines are cut by a third line, the sum of the measures of the interior angles on the same side of the transversal is 180°. In the figure,

$$c + w = 180 \quad \text{and} \quad d + x = 180.$$

Angle Relationships



1. The sum of the measures of the interior angles of a triangle is 180°. In the figure above,

$$x = 70 \text{ because } 60 + 50 + x = 180.$$

2. When two lines intersect, vertical angles are congruent. In the figure,

$$y = 50.$$

3. A straight angle measures 180°. In the figure,

$$z = 130 \text{ because } z + 50 = 180.$$

4. The sum of the measures of the interior angles of a polygon can be found by drawing all diagonals of the polygon from one vertex and multiplying the number of triangles formed by 180°.

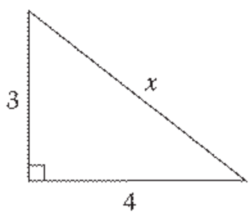
Since this polygon is divided into 3 triangles, the sum of the measures of its angles is $3 \times 180°$, or 540°.

Unless otherwise noted in the SAT, the term "polygon" will be used to mean a convex polygon, that is, a polygon in which each interior angle has a measure of less than 180°.

A polygon is "regular" if all its sides are congruent and all its angles are congruent.

Side Relationships

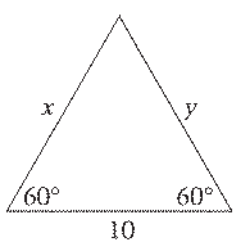
1. Pythagorean Theorem: In any right triangle, $a^2 + b^2 = c^2$, where c is the length of the longest side and a and b are the lengths of the two shorter sides.



To find the value of x, use the Pythagorean Theorem.

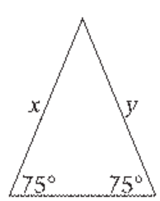
$$x^2 = 3^2 + 4^2$$
$$x^2 = 9 + 16$$
$$x^2 = 25$$
$$x = \sqrt{25} = 5$$

2. In any equilateral triangle, all sides are congruent and all angles are congruent.

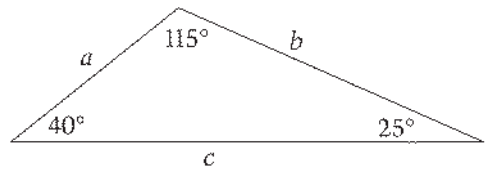


Because the measure of the unmarked angle is 60°, the measures of all angles of the triangle are equal; therefore, the lengths of all sides of the triangle are equal: $x = y = 10$.

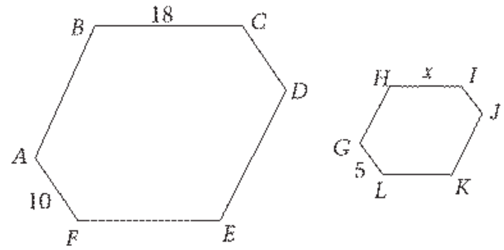
3. In an isosceles triangle, the angles opposite congruent sides are congruent. Also, the sides opposite congruent angles are congruent. In the figures below, $a = b$ and $x = y$.



4. In any triangle, the longest side is opposite the largest angle, and the shortest side is opposite the smallest angle. In the figure below, $a < b < c$.



5. Two polygons are *similar* if and only if the lengths of their corresponding sides are in the same ratio and the measures of their corresponding angles are equal.



If polygons $ABCDEF$ and $GHIJKL$ are similar, then $\overline{AF}$ and $\overline{GL}$ are corresponding sides, so that

$$\frac{AF}{GL} = \frac{10}{5} = \frac{2}{1} = \frac{BC}{HI} = \frac{18}{x}.$$ Therefore, $x = 9 = HI$.

Note: $\overline{AF}$ means the line segment with endpoints A and F, and AF means the length of $\overline{AF}$.

Area and Perimeter

Rectangles

Area of a rectangle = length × width = $\ell \times w$

Perimeter of a rectangle = $2(\ell + w) = 2\ell + 2w$

Circles

Area of a circle = πr^2 (where r is the radius)

Circumference of a circle = $2\pi r = \pi d$ (where d is the diameter)

Triangles

Area of a triangle = $\frac{1}{2}$(base × altitude)

Perimeter of a triangle = the sum of the lengths of the three sides

Triangle inequality: The sum of the lengths of any two sides of a triangle must be greater than the length of the third side.

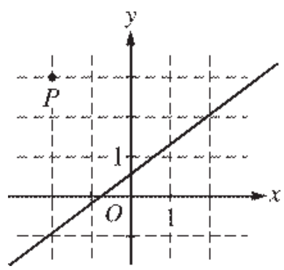
Volume

Volume of a rectangular solid (or cube) $= \ell \times w \times h$
(ℓ is the length, w is the width, and h is the height)

Volume of a right circular cylinder $= \pi r^2 h$
(r is the radius of the base, and h is the height)

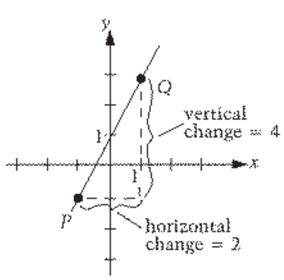
Be familiar with the formulas that are provided in the Reference Information included with the test directions. Refer to the test directions in the sample test in this publication.

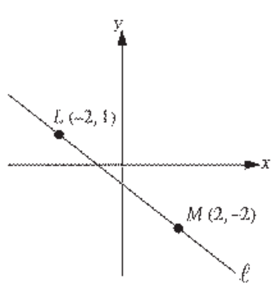
Coordinate Geometry



1. In questions that involve the x- and y-axes, x-values to the right of the y-axis are positive and x-values to the left of the y-axis are negative. Similarly, y-values above the x-axis are positive and y-values below the x-axis are negative. In an ordered pair (x, y), the x-coordinate is written first. Point P in the figure above appears to lie at the intersection of gridlines. From the figure, you can conclude that the x-coordinate of P is -2 and the y-coordinate of P is 3. Therefore, the coordinates of point P are $(-2, 3)$. Similarly, you can conclude that the line shown in the figure passes through the point with coordinates $(-2, -1)$ and the point $(2, 2)$.

2. Slope of a line $= \dfrac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$





$$\text{Slope of } \overline{PQ} = \frac{4}{2} = 2$$

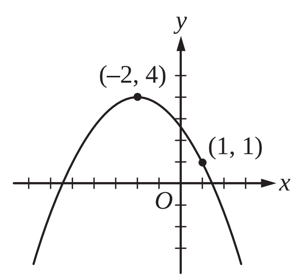
$$\text{Slope of } \ell = \frac{1 - (-2)}{-2 - 2} = -\frac{3}{4}$$

A line that slopes upward as you go from left to right has a *positive* slope. A line that slopes downward as you go from left to right has a *negative* slope. A horizontal line has a slope of zero. The slope of a vertical line is undefined.

Parallel lines have the same slope. The product of the slopes of two perpendicular lines is -1, provided the slope of each of the lines is defined. For example, any line perpendicular to line ℓ above has a slope of $\frac{4}{3}$.

The equation of a line can be expressed as $y = mx + b$, where m is the slope and b is the y-intercept. Since the slope of line ℓ is $-\frac{3}{4}$, the equation of line ℓ can be expressed as $y = -\frac{3}{4}x + b$. Since the point $(-2, 1)$ is on the line, $x = -2$ and $y = 1$ must satisfy the equation. Hence, $1 = \frac{3}{2} + b$, so $b = -\frac{1}{2}$, and the equation of line ℓ is $y = -\frac{3}{4}x - \frac{1}{2}$.

3. A quadratic function can be expressed as $y = a(x - h)^2 + k$ where the vertex of the parabola is at the point (h, k) and $a \neq 0$. If $a > 0$, the parabola opens upward; and if $a < 0$, the parabola opens downward.



The parabola above has its vertex at $(-2, 4)$. Therefore, $h = -2$ and $k = 4$. The equation can be represented by $y = a(x + 2)^2 + 4$. Since the parabola opens downward, we know that $a < 0$.
To find the value of a, we also need to know another point on the parabola. Since we know the parabola passes through the point $(1, 1)$, $x = 1$ and $y = 1$ must satisfy the equation. Hence, $1 = a(1 + 2)^2 + 4$, so $a = -\frac{1}{3}$. Therefore, an equation for the parabola is $y = -\frac{1}{3}(x + 2)^2 + 4$.

Data Analysis, Statistics, and Probability

Measures of Center

An **average** is a statistic that is used to summarize data. The most common type of average is the **arithmetic mean**. The average (arithmetic mean) of a list of n numbers is equal to the sum of the numbers divided by n.

For example, the mean of 2, 3, 5, 7, and 13 is equal to

$$\frac{2+3+5+7+13}{5} = 6.$$

When the average of a list of n numbers is given, the sum of the numbers can be found. For example, if the average of six numbers is 12, the sum of these six numbers is 12×6, or 72.

The **median** of a list of numbers is the number in the middle when the numbers are ordered from greatest to least or from least to greatest. For example, the median of 3, 8, 2, 6, and 9 is 6 because when the numbers are ordered, 2, 3, 6, 8, 9, the number in the middle is 6. When there is an even number of values, the median is the same as the mean of the two middle numbers. For example, the median of 6, 8, 9, 13, 14, and 16 is the mean of 9 and 13, which is 11.

The **mode** of a list of numbers is the number that occurs most often in the list. For example, 7 is the mode of 2, 7, 5, 8, 7, and 12. The list 2, 4, 2, 8, 2, 4, 7, 4, 9, and 11 has two modes, 2 and 4.

Note: On the SAT, the use of the word *average* refers to the arithmetic mean and is indicated by "average (arithmetic mean)." An exception is when a question involves average speed (see page 15). Questions involving median and mode will have those terms stated as part of the question's text.

Probability

Probability refers to the chance that a specific outcome can occur. When outcomes are equally likely, probability can be found by using the following definition:

$$\frac{\text{number of ways that a specific outcome can occur}}{\text{total number of possible outcomes}}$$

For example, if a jar contains 13 red marbles and 7 green marbles, the probability that a marble selected from the jar at random will be green is

$$\frac{7}{7+13} = \frac{7}{20} \text{ or } 0.35$$

Note: The phrase *at random* in the preceding example means that each individual marble in the jar is equally likely to be selected. It does not mean the two colors are equally likely to be selected.

If a particular outcome can never occur, its probability is 0. If an outcome is certain to occur, its probability is 1. In general, if p is the probability that a specific outcome will occur, values of p fall in the range $0 \leq p \leq 1$. Probability may be expressed as either a decimal, a fraction, or a ratio.