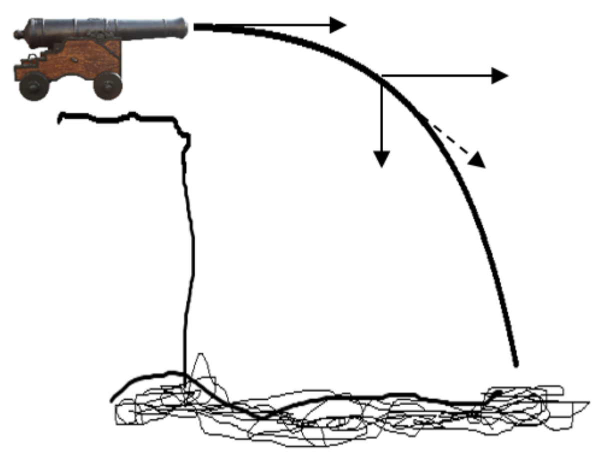
Unit 3 Parent Guide: Application of Forces

Part I: Two-Dimensional Motion

Unit 1 addressed speed, velocity and acceleration in the horizontal or vertical planes. Two-dimensional motion occurs when an object in motion has both vertical and horizontal components. When the motion of an object having these two components is only being influenced by the force of gravity (in the vertical direction), it is called *projectile motion*. So, once the object is set in motion, only gravity will influence the path it travels and the changes in position and velocity that will occur. Since gravity is a force that acts only vertically (downward), it only affects the vertical component, not the horizontal component of a projectile.

The most basic projectile is an object that only has vertical motion as mentioned in Case 3 above. Whether dropped from rest or launched at a 90^0 angle (straight up), this type of projectile's motion does not have a horizontal component. This motion is similar to linear acceleration presented above in this document and so the motion diagrams, motion graphs, and kinematics equations are all similar.

The other two types of projectile motion are 2-dimensional. There is simultaneously horizontal and vertical motion. One type of projectile motion occurs when an object is launched horizontally from some height above the ground. The *trajectory* (path) of this type of motion would have the shape of a *half-parabola*. As the projectile is fired (launched), its horizontal velocity does not change; there is no horizontal acceleration. The horizontal distance covered by the projectile is called the *range* of the object.

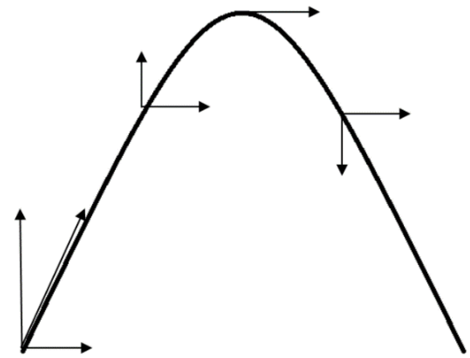


The vertical velocity does change at a rate equal to the acceleration due to gravity, 9.8 m/s^2. When an object is launched horizontally, such as from this cannon, initially it has no vertical velocity, only horizontal velocity. The initial vertical velocity is zero and will change (increase in the negative direction) as long as it is freely falling. It is also important to remember, *horizontal motion has no effect on vertical motion.* The horizontal speed of a cannonball does not affect the vertical motion at all, so horizontal velocity will remain constant while vertical velocity changes. Notice in these diagrams how the horizontal arrow is constant while the vertical arrow is changing.

An excellent animantion and illustration of this motion can be found at horizontally launched projectile at physicsclassroom.com. Once launched, the projectile has both vertical and horizontal velocity and the combination of these two components is represented by the parabolic path. The combination of these two vectors at any point is called the resultant, determined using the pythagorean theorem and shown in the diagram above as a dotted arrow.

The third type of projectile motion is when an object is launched up at an angle. Many of the same principles that applied to horizontally launched objects will also apply to non-horizontally launched objects. These factors include the absence of horizontal acceleration, the presence of vertical acceleration and the creation of a trajectory that is parabolic.

There are two significant differences between the two types of 2-D projectile motion. First, in addition to an initial horizontal velocity, the projectile also has an initial vertical velocity. Yet, just as with horizontally launched projectiles, the horizontal velocity will remain unchanged throughout its trajectory since there is no horizontal acceleration. Since there is a vertical acceleration due to the force of gravity, the initial vertical velocity will decrease by 9.8 m/s^2 just as mentioned before.



Part II: Applications of Forces

Gravitational Force: All objects are attracted to every other object with a gravitational force that is directly proportional to the product of the their masses and inversely proportional to the square of the distance they are separated at their centers. This is known as Newton's Law of Gravitation.

$$F_g = \frac{Gm_1m_2}{r^2}$$

The quantity *G* is the gravitational constant whose value is $6.67 \times 10^{-11}\,\frac{m^3}{kg\,s^2}$. This force is very weak but is noticeable for large objects like stars and planets. The force that we commonly call **weight** is really the gravitational force near the surface of the earth. Plugging in the values for the mass and radius of the earth, and considering that they remain constant near the surface, the equation above simplifies to:

$$F_g = mg$$

Where the quantity *g* is the gravitational field strength or equivalently, the acceleration due to gravity, whose value at Earth's surface is approximately 9.81 m/s^2. So the force we call weight is simply the gravitational force at the earth's surface. For convenience, the value of g is often rounded to 10 m/s^2.

Normal Force: When two objects are pushed together, their surfaces exert force on one another which is called normal force. This is often observed when one object is supported by another object such as a floor, table, road, or ramp. Here the meaning of the word normal is perpendicular. A cart resting or moving along a horizontal floor is supported by an upward normal force. A cart sliding down a ramp is pushed by the ramp with a normal force whose direction is perpendicular to and away from the surface of the ramp. When the surface is static, the normal force is often balanced by other forces such as gravity or other force so that there is no net force along the direction of the normal force. The result of this balance is that the object does not accelerate into or away from the surface, but may rest on the surface or slide along it.

Friction Force: Friction is the name of the force that may be present when the surfaces of two objects are in contact. Friction results when the two surface slide (or attempt to slide) along one another, so as to resist the relative motion (or intended motion) of the surfaces. The direction of the friction force is always perpendicular to the normal force between the surfaces, so that the friction force is parallel to the plane of the surface.

Kinetic Friction: When a box is being dragged along a floor, the friction force on the box results from the surface of the floor pushing on the surface of the bottom of the box in the opposite of the direction that the box is moving. This type of friction is known as kinetic friction because it opposes the actual motion of the two forces sliding along one another. This is the force that is commonly seen to “slow down” objects when other forces are no longer applied.

Static Friction: When a box is at rest on the floor, but is being pushed and not moving, we say that there is static friction between the two surfaces. The static friction opposes the intended motion of the surfaces, to keep them at rest. Static friction is also known as “traction” for car tires and even soles of shoes. When you take a step forward, static friction is actually preventing your back foot from sliding backward. The floor is actually pushing your back foot forward in an effort to oppose the motion of it sliding backward. Vehicles move similarly. The road pushes a car tire forward to prevent it from spinning in place. The bottom of a tire would move backward if there were no friction. Static friction prevents the tire from moving backward by pushing it forward.

Friction is a force that depends on two quantities: the normal force, and the types of surface materials. The normal force was explained above. To quantify the surface type, we use the ratio of friction to normal force, as in the relationship below:

$$\mu = \frac{friction\ force}{normal\ force}$$

The quantity μ is known as the *coefficient of friction*. The COF is a ratio of these two forces so it is unitless, but it is a measure of the ability of two surfaces to produce friction. Values of μ are small, typically less than 1. Surfaces with higher values for μ, such as rubber on concrete, potentially can produce higher amounts of friction. Lower values such as that observed for steel on ice have lower amounts of friction. Typically the value of the coefficient of static friction is greater than the value of coefficient of kinetic friction for a given pair of surfaces.

Tension Force: Tension is the name of a force exerted on an object being pulled by a rope, string, cable, or chain. Here the application of force is along the rope by the action of pulling. There are some special considerations for tension. In an ideal string, tension force is the same on both ends of the string. That means that two objects connected by a string each experience the same pulling force by the string, as if the objects were pulling one another. In fact, even when a string changes direction by moving around a frictionless corner or pulley with no mass, the tension is the same on both ends. This means that strings can change the direction of pulling force.

Spring Force: Springs exert force as they are stretched or compressed from their relaxed (equilibrium) position. The force exerted by a spring on an object is directly proportional to the displacement of the spring from its equilibrium position. This relationship is known as Hooke’s Law:

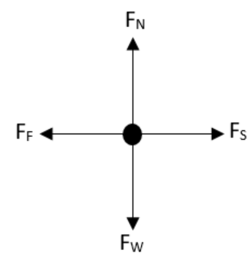
$$F_S = -kx$$

where F_s is the spring force, x is the displacement from equilibrium and k is known as the spring constant, a measure of the stiffness of the spring. Spring force is known as a restoring force because the direction of the force is always opposite of the direction of the displacement. When you compress a spring, the spring pushes back toward equilibrium.

Free-Body Diagrams

It is useful in the analysis of forces to draw a diagram that shows individual forces as arrows acting on a single object. This diagram is called a free-body diagram. The diagrams shown in Scenarios A, B, & C are not free-body diagrams. Take another look at Scenario A.

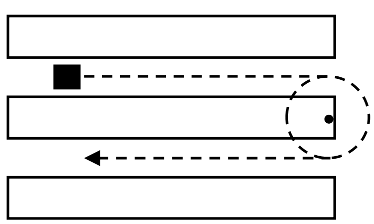
A free-body diagram of the cart should show all of the individual forces that at on the cart. That should include the force applied by the shopper (F_s to the right), but also the weight force (F_W down) the normal force (F_N upward) and friction (F_F leftward). The diagram should something like like this:



Circular Motion

Thus far in this document the discussion of forces has been limited to forces that are aligned along a straight line, and that may result in acceleration along that line. Objects moving along a circle still follow Newton's 3 Laws, but with one change, the direction of force is continually changing.

Viewed from above, a shopper pushes a cart (the black box shown) in a straight line down the grocery aisle. At the end of the aisle, the cart makes a right turn along a circular path to the adjacent aisle. In order to make this turn, the direction of the cart must continually change. So the cart is accelerating since it is changing direction. But the direction of this acceleration is inward, toward the center of the circle. The cart does not move toward the center, but it *accelerates* toward the center.



Any object whose motion is along a circle must be accelerating toward the center of the circle. The direction of the acceleration is called *centripetal* (center seeking). Centripetal acceleration a_c is given by the equation:

$$a_c = \frac{v^2}{r}$$

where v is the speed of the object and r is the radius of the circle. Since acceleration is caused by force, the centripetal acceleration must be caused by a net force along the same direction as the acceleration, toward the center. Any force that causes an object to move in a circular motion is called centripetal force. Centripetal Force is not a unique special kind of force. Any of the forces mentioned in this document can serve to act as a centripetal force.

Common examples of motion along a circular path:

Orbital motion: As the earth moves in a (nearly) circular path around the sun, it is pulled by the sun's gravity. The sun is (nearly) at the center of the earth's path, which means that the earth is pulled toward the center of a circle. So the sun's gravity acts as a centripetal force which causes the earth (and many other objects) to move along a circular path. Note: the earth's orbit is elliptical but it is nearly circular. For the purposes of this discussion, it's helpful to call it a circle.

A car turning: Friction between the road and tires allows a car to turn. Turning the steering wheel is useless if there is no friction. The direction of the friction is toward the center of the turn.

Roller coaster: When the car on a roller coaster track enters a loop, it is pushed by the track in order to change direction. While upside down, the track and earth's gravity both push downward on the car, which is toward the center of the circle. At all parts of the loop, the track pushes toward the center. Because the car doesn't move at a constant speed along the track, the centripetal force is not constant, but there must be a component of force toward the center of the circle.

It is useful in the discussion of circular motion to revisit Newton's 1^{st} law. If the cart mentioned above were "let go" at point A in the diagram below, it would follow the path of the solid black line shown (downward on the diagram). Letting go of the cart means that the force acting on the cart is removed. Now the cart is simply an object in motion that will keep the same motion it had the moment before the force was removed.